$$\frac{1}{x^2-x}=\frac{1}{x(x-1)}=\frac{1}{x-1}-\frac{1}{x}$$

$$(\ldots)\frac{1}{x^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3} \qquad \frac{Cx+D}{x^2+1}$$

$$\frac{P(x)}{Q(x)}=\frac{A_n}{(x-a)^n}+\frac{A_{n-1}}{(x-a)^{n-1}}+ \text{—} +\frac{A_1}{(x-a)^1}+$$

$$+\frac{B_m}{(x-b)^m}+\frac{B_{m-1}}{(x-b)^{m-1}}+ \text{—} +\frac{B_1}{(x-b)^1}+$$

$$+$$

$$|$$

$$+$$

$$\frac{C_s x+D_s}{(p_1x^2+q_1x+r_1)^s}+\frac{C_{s-1}x+D_s}{(p_1x^2+q_1x+r_1)^{s-1}}+ \text{—} +\frac{C_1x+D}{p_1x^2+q_1x+r_1}+$$

$$+$$

$$|$$

Pří:

$$\int \frac{1}{x^2-x}\,dx = \int \frac{1}{x(x-1)}\,dx$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} = \frac{A(x-1)+Bx}{x(x-1)}$$

$$1 = A(x-1)+Bx = Ax - A + Bx = (A+B)x - A$$

$$0 = A+B$$

$$1 = -A$$

$$1 = A(x-1)\underline{+Bx}$$

$x=1$ $\quad 1 = B$

$x=0$ $\quad 1 = -A$

$$\frac{x+2}{x(x-2)(x+1)} = \frac{-1}{x} + \frac{2/3}{x-2} + \frac{1/3}{x+1}$$

$$x+2 = A(x-2)(x+1) + B(x+1)x + Cx(x-2)$$

$x=0 \quad 2 = A\cdot(-2)\cdot 1 \quad A=-1$

$x=2 \quad 4 = B\cdot 3\cdot 2 \quad B = 2/3$

$x=-1 \quad 1 = C\cdot(-1)\cdot(-3) \quad C = \frac{1}{3}$

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$x=-1 \quad 1 = A \cdot 3 \quad A = 1/3$

$x=0 \quad 1 = A + C \cdot 1 \quad C = 2/3$

$x=1 \quad 1 = A + (B+C) \cdot 2$

$1 = \frac{1}{3} + \left(B + \frac{2}{3}\right) 2$

$2 = 6B + 4 \qquad B = -\frac{1}{3}$

$$\int \frac{1}{x^3+1}\,dx = \int \underbrace{\frac{1}{3}\frac{1}{x-1}}_{\frac{1}{3}\ln|x-1|} + \frac{1}{3}\,\frac{-x+2}{x^2-x+1}\,dx$$

$$\int \frac{-x+2}{x^2-x+1}\,dx = -\frac{1}{2}\int \frac{2x-1-3}{x^2-x+1}\,dx =$$

$$= \underbrace{-\frac{1}{2}\int \frac{2x-1}{x^2-x+1}\,dx}_{-\frac{1}{2}\ln(x^2-x+1)} - \frac{1}{2}\int \frac{-3}{x^2-x+1}\,dx$$

$$\int \frac{dx}{x^2-x+1} = \int \frac{dx}{\underbrace{x^2-x+\frac{1}{4}}_{(x-\frac{1}{2})^2}+\frac{3}{4}} = \int \frac{dx}{(x-\frac{1}{2})^2+\frac{3}{4}} = \left| \begin{matrix} t = x - \frac{1}{2} \\ dt = dx \end{matrix} \right| =$$

$$= \int \frac{dt}{t^2+\frac{3}{4}} = \frac{4}{3}\int \frac{dt}{\frac{4}{3}t^2+1} = \frac{4}{3}\int \frac{dt}{(\frac{2}{\sqrt{3}}t)^2+1} =$$

$$= \left| \begin{matrix} z = \frac{2}{\sqrt{3}}t \\ dz = \frac{2}{\sqrt{3}}dt \end{matrix} \right| = \frac{4}{3}\int \frac{\frac{\sqrt{3}}{2}\,dz}{z^2+1} = \frac{2\sqrt{3}}{3}\arctan z$$

$$\frac{2\sqrt{3}}{3}\arctan\frac{2t}{\sqrt{3}}$$

$$\frac{2\sqrt{3}}{3}\arctan\frac{2x-1}{\sqrt{3}}$$

$$\frac{4x^3+4x-4}{x(x^2+2)^2} = \frac{A}{x} + \frac{Bx+C}{(x^2+2)^2} + \frac{Dx+E}{x^2+2}$$

$$4x^3+4x-4 = A(x^2+2)^2 + (Bx+C)x + (Dx+E)(x^2+2)x$$

$x=0 \quad -4 = 4A \quad A=-1$

$x=1 \quad 4 = 9A + (B+C) + (D+E)\cdot 3$

$13 = B+C+3D+3E$

$x=-1 \quad -12 = 9A - (C-B) - (E-D)\cdot 3$

$-3 = B-C+3D-3E$

$x=2 \quad 36 = -36 + 2(2B+C) + 2(2D+E)\cdot 6$

$72 = 4B+2C+24D+12E$

$x=-2 \quad -44 = -36 - 2(C-2B) - 2(E-2D)\cdot 6$

$-8 = 4B-2C+24D-12E$

$13 = B+C+3D+3E$

$-3 = B-C+3D-3E$

$72 = 4B+2C+24D+12E$

$-8 = 4B-2C+24D-12E$

$13 = B+C+3D+3E$

$5 = B \quad +3D$

$36 = 2B+C+12D+6E$

$8 = B \quad +6D$

$13 = B+C+3+3E$

$-3 = \quad -3D \qquad D=1$

$36 = 2B+C+12+6E$

$8 = B \quad +6 \qquad B=2$

$8 = \quad +C+3E$

$20 = \quad +C+6E$

$12 = \quad 3E \qquad E=4$

$C=-4$

$$\frac{4x^3+4x-4}{x(x^2+2)^2} = \frac{-1}{x} + \frac{2x-4}{(x^2+2)^2} + \frac{x+4}{x^2+2}$$