Opava, 25 October 2004 – 29 October 2004
Slezská univerzita v Opavě
Opava, Česká republika

Perturbations of monotonic mod one transformations

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Abstract. We call a map $T : [0,1] \to [0,1]$ a monotonic mod one transformation, if there exists a continuous strictly increasing function $f : [0,1] \to \mathbb{R}$, such that $Tx = f(x) - \big[f(x)\big]$ holds for all $x \in (0,1) \setminus f^{-1}(\mathbb{Z})$, where $[y]$ denotes the largest integer smaller than or equal to y. For simplicity we write $Tx = f(x) \pmod 1$. Maps of these type occur in some applications, and also in number theory.
Next some different types of perturbations of monotonic mod one transformations are discussed. In some sense the most natural type of perturbations is given by the following topology on the class of monotonic mod one maps: given $\varepsilon > 0$, $\widetilde{T}$ and T are called ε-close, if there exist $\widetilde{f}$ and f with $\widetilde{T}x = \widetilde{f}(x) \pmod 1$, $Tx = f(x) \pmod 1$ and $\|\widetilde{f} - f\|_\infty < \varepsilon$.
The stability of monotonic mod one transformations under small perturbations in the topology described above is investigated. An example shows that the topological entropy is not upper semi-continuous in general. However, the topological entropy is always lower semi-continuous, and introducing a certain oriented graph $(\mathcal{G}', \to)$ one can give upper bounds for the jumps up of the topological entropy. It turns out that $h_{\mathrm{top}}(T) > 0$ implies that the topological entropy is continuous at T. Conditions implying the continuity of the measure of maximal entropy are also given. As an example shows the measure of maximal entropy needs not be continuous, even if $h_{\mathrm{top}}(T) > 0$ and T has a unique measure of maximal entropy.
Given a continuous function $g : [0,1] \to \mathbb{R}$, one can give upper bounds for the jumps up of the topological pressure. In general, the topological pressure needs not be lower semi-continuous. Finally, the topological pressure is upper semi-continuous at T for every continuous function $g : [0,1] \to \mathbb{R}$, if and only if 0 is not periodic or 1 is not periodic. This means that jumps up of the topological pressure at T can only occur, if both 0 and 1 are periodic.

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