Contents

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LIFE AT THE SILESIAN UNIVERSITY

The Academic Year

Winter semester:	October – December
Exams:	January
Summer semester:	February – May
Exams:	June

Attention! This dates change every year, the up-to-date and exact ones can be found on the web pages of the Mathematical Institute http://www.math.slu.cz, in the study programme. Alternatively you can ask the co-ordinator.

Socrates Institutional Coordinator

	Petra Auerová
Address:	Silesian University in Opava
	Bezručovo nám. 13
	746 01 Opava
	Czech Republic
E-mail:	petra.auerova@slu.cz
Tel:	+420 553 684 270
Fax:	+420 553 684 262
WWW:	http://www.slu.cz

Socrates Departmental Coordinator

	Jana Šindlerová
Address:	Mathematical Institute
	Silesian University in Opava
	Bezručovo nám. 13
	746 01 Opava
	Czech Republic
E-mail:	Jana.Sindlerova@math.slu.cz
Tel:	+420 553 684 360
Fax:	+420 553 715 029
WWW:	http://www.math.slu.cz

Living Expenses

Accommodation	60 EUR	(about 1800 CZK)
Food, clothing	120 EUR	(about 3500 CZK)
Transport	10 EUR	(about 250 CZK)
Books	15 EUR	(about 450 CZK)
Other expenses	50 EUR	(about 1500 CZK)
Total	255 EUR	(about 7500 CZK per month)

Insurance and Medical Care

In the Czech Republic health insurance valid within the EU is mostly recognised. Nevertheless, you should check with your insurance office whether this is the case of your particular health insurance. You should also take an additional insurance covering hospital care, if necessary. This does not apply to British and Greek students (all health care is covered by bilateral agreements).

Insurance providers

Allianz
Nám. Republiky 11

Česká pojišťovna
Hrnčířská 1

Generalli
Nákladní 41

Kooperativa
Sady Svobody 4

Computers at the Silesian University

Students can freely enter several computer laboratories, equipped with Macintosh and PC. All of them are connected to the Internet.

The computer laboratories have got their web pages, where you can find all information on the work of the laboratories: http://www.labs.math.slu.cz

The PC computer laboratories are administered by the Faculty of Philosophy and Science.

Libraries

Mathematical Institute library is well supplied with mathematical literature, including more than 50 international mathematical journals. It is freely available to students. The list of available journals contains, e.g.

Acta Applicandae Mathematicae
Amer. Math. Monthly
Ann. Global Anal. Geom.,
Ergodic Theory Dynam. Systems
J. Geom. Phys.
J. Math. Anal. Appl.
Math. Proc. Cambridge Philos. Soc.
Math. Reviews
Proc. Amer. Math. Soc.
Trans. Amer. Math. Soc.

For complete list see http://www.math.slu.cz/knihovna/casopisy.php

Information on Opava

Opava is an ancient town and historical centre of the Czech part of the Silesian region. The foundation of the Opavian principality is connected with the Přemyslide dynasty. From 1850 to 1928, Opava was the capital of the Silesian region. Opava of today is very much a cultural city. We can find there a great number of historical monuments, beautiful parks with pleasant quiet corners spread throughout the city centre. The rich cultural life owes to the Silesian Theatre (both opera and drama), the Marionette Theatre, the House of Art, numerous private galleries, exhibitions at the Silesian Museum along with concert and dance halls.

The sporting life is practised at the Tyrš Stadium and Winter Stadium, tennis courts, and riding-schools. Every year numerous activities that exceed the regional borders take place there. The most important among them is the Bezruč Opava Festival, the Festival of Spiritual Music, the Young Organ Players Competition and the worldwide Orchestra Conducting Courses. Opava is a seat of one of the youngest universities in the Czech Republic-the Silesian University, whitch was established in 1990.

Opava is not only a beautiful city, but also a very friendly one. We wish all the visitors of the city a pleasant stay and many wonderful experiences.

For more information and photos see official web-page of the City of Opava:
http://www.opava-city.cz

Detailed Information about Accommodation Possibilities

The number of rooms in student dormitories of the Silesian University in Opava is limited. Erasmus students who submit their applications early are given priority. The remaining rooms are allocated on a „first-come, first-served“ basis.

Usually, Erasmus students accepted for study at the Silesian University in Opava will be guaranteed accommodation:

Palhanec
Residential Halls,
746 01 Opava

Residential Halls
Komárovská Street 25
746 01 Opava

Accommodation is in well-equipped three-bed study rooms sharing a kitchen and a bathroom. All study bedrooms have bed and bedding. Each kitchen is equipped with an electric or gas cooker, a fridge, a table and chairs.

University Club of the Silesian University

The tea-house Bludný kámen (situated in the historical centre of the town, on Dolní náměstí) has a position of a unique cultural phenomenon since its establishment. Its activities are intended not only for the university students, but support cultural life, especially in the alternative culture. The tea-room is opened almost every day and artists, scientists or other famous people hold meetings on many evenings.

Association of Students and Friends of the Silesian University

The association holds the University balls and rag-days (Majáles), it financially helps students and teachers, publishes lecture notes and so on. All this is done by means of a development study fund.

Cultural Possibilities in Opava

Theatres

Silesian Theatre
Horní náměstí 13

Marionette Theatre
Husova 17

Petr Vaněk Harlequin Theatre
Kateřinky, Vrchní 41

Cinemas

Mír
Kolářská 5

Elektra (+ Film Club)
Havlíčkova 8

Museums

Silesian Regional Museum in Opava

- the Main Exhibition Building, U muzea 1
- the Memorial of Petr Bezruč, Ostrožná 35
- the Arboretum, Nový Dvůr by Opava
- the Memorial of the World War Two, Hrabyně
- the Memorial of the Former Czechoslovak Fortification, Milostovice

Galleries and Exhibition Halls

House of Art
Pekařská 12

Jakob's Gallery
Dolní náměstí 13

BT Gallery – Wood
Otická 11

Castles

Hradec upon Moravice
Raduň
Kravaře

Libraries

Petr Bezruč District Library

- main building, Nádražní okruh 27. http://www.okpb.cz
- Local Office Kateřinky, Šrámkova 6
- Local Office Kylešovice, Liptovská 21
- Reading Room, Minorite Monastery building, Masarykova 41

Sport Possibilities

Fitness centres

- Bavaria fitness: Kylešovice, Hlavní 68
- Pepa Sport: Zámecký okruh 8
- Studio Relax: Zámecký okruh 4
- Vitalklub: Provaznická 2

Tennis & Squash

- Tennis courts, Hradecká 1
- Tennis courts, Nerudova 25
- Tennis area, Wolkerova 1a
- AB Squash Centre, Fügnerova 52
- Squash Club: Solná 23

Swimming Pool
Jaselská 35

Horse Riding
Horse Riding Club Opava: Kateřinky, Rolnická 120
School Farm Riding Hall: Englišova 526

Health Care

State Silesian Hospital in Opava
Olomoucká 86

State Silesian Hospital
Popská 9

Clinic Opava-Předměstí
Nám. Slezského odboje

Psychiatric Medical Institution
Olomoucká 89

Financial Institutions

Česká spořitelna
Nám. Republiky 15

ČSOB
Ostrožná 17
Hrnčířská 6

GE Capital Bank
Ostrožná 31

Komerční banka
Hrnčířská 2
Ostrožná 46

Union banka
U pošty 8

Živnostenská banka
Nám. Republiky 8

THE SILESIAN UNIVERSITY IN OPAVA

The Silesian University in Opava, (hereafter Silesian University or in short SU), is one of the youngest universities in the Czech Republic. The Masaryk University in Brno was appointed the legal authority responsible for the preparations for the founding of the Silesian University. On 17th November 1990 two faculties were established following the decision of the Academic Senate of the Masaryk University in Brno: the Faculty of Philosophy in Opava (with both humanities and sciences) and the School of Business Administration in Karviná.

On 8th October 1990 study courses on both faculties commenced. According to the Česká národní rada (Czech National Council) act from 9th July 1991, which came into force on 28th September 1991, the Silesian University with the seat in Opava was established.

Within the Faculty of Philosophy, the Institute of Mathematics was formed. The Faculty of Philosophy was renamed Faculty of Philosophy and Science on 30th July 1992.

The Institute development was very quick. Together with the bachelor and master studies, (in scientific as well as in teaching specialisation), PhD-studies of Mathematical Analysis could also begin in 1994. Since 1995 the Institute has had the right to confer the Associate Professor`s and Professor`s degrees and since 1997 the PhD-studies in the second branch, (Geometry and Global Analysis), have been founded. On 1st January 1997 the Institute of Mathematics was divided into two parts: the Institute of Mathematics and the Institute of Informatics.

On 1st January 1999 the Mathematical Institute of the Silesian University was formed out of the former Institute of Mathematics of the Faculty of Philosophy and Science. The Mathematical Institute is a University institute according to the § 34 of the Universities Law, No. 111/1998. The Mathematical Institute is responsible for the bachelor, master and doctoral studies in the “Mathematics” study programme. In the co-operation with the Faculty of Philosophy and Science of the SU, it provides the mathematical part of the study of mathematics for secondary school teachers and courses on mathematics for the students of Physics, Computer Science etc. The Mathematical Institute has a right to confer the Doctor Rerum Naturalium, Doctor of Philosophy, Associate Professor`s and Professor`s degrees in the mathematical branches.

Rector:

Zdeněk Jirásek

Vice-rectors:

František Koliba – science and foreign affairs
Rudolf Žáček – education and social issues
Eva Wagnerová – development

Bursar:

Jaroslav Kania

Office of the Rector of the Silesian University in Opava

Address: Bezručovo nám. 13
746 01 Opava
Czech Republic
Tel.: +420 553 684 272
Fax: +420 553 718 019
E-mail: rektorat@slu.cz
WWW: http://www.slu.cz

MATHEMATICAL INSTITUTE IN OPAVA

Director:

Jaroslav Smítal

Deputy Directors:

Marta Štefánková – science and foreign affairs
Michal Marvan – information technologies
Kristína Smítalová – study affairs

Contact:

Address: Mathematical Institute in Opava
Silesian University in Opava
Bezručovo nám. 13
746 01 Opava
Tel.: +420 553 684 341
Fax: +420 553 684 217
E-mail: math@math.slu.cz
WWW: www.math.slu.cz

Administration Office

Jiřina Böhmová – head of the administration office
Jana Šindlerová – official responsible for study and economic matters
Jana Malíčková – librarian
Aleš Rod – Applied Mathematics in Risk Management Project Coordinator
Michal Málek – alternative community service to military service

Department of Applied Mathematics

Teachers hold classes of bachelor and master studies. They also cover running of the computing laboratory.

Assistant Professors

Jaromír Sýkora – head of the department
Libuše Hozová (part-time position)
Vladimír Sedlář
Martin Snethlage

Computing Support:

Michal Mikláš
Petr Kolovrat (part-time position)
Jan Morisch (part-time position)

Department of Global Analysis

Teachers of the department hold classes of bachelor, master and doctoral studies and supervise students of the doctoral studies in mathematics. In the co-operation with the Faculty of Science of the Masaryk University in Brno they take part in the education of PhD-students in physics. They carry out scientific research in the global analysis, differential geometry, mathematical physics and related fields.

Associate Professors

Tomáš Kopf – head of the department
Lubomír Klapka
Olga Krupková
Michal Marvan

Assistant Professors

Oldřich Stolín
Artur Sergyeyev

PhD-students (full-time)

Petr Chládek
Jan Kotůlek
Milan Pobořil
Aleš Rod
Dana Smetanová
Martin Swaczyna
Jana Šeděnková
Petr Volný

Department of Mathematical Analysis

Teachers hold classes of bachelor, master and doctoral studies. They also supervise students of doctoral studies. They carry out scientific research in mathematical analysis and dynamical systems.

Professors:

Jaroslav Smítal
Vladimir Iosifovič Averbuch

Associate Professors:

Kristína Smítalová – head of the department

Assistant Professors:

Karel Hasík
Zdeněk Kočan
Jana Kopfová
Marta Štefánková

PhD-students (full-time)

Lenka Čelechovská
Jiří Kupka
Marek Lampart
Michal Málek
Jan Melecký
Petra Šindelářová

Academic Council of the Mathematical Institute

Chairman:

Jaroslav Smítal

Internal Members:

Vladimir Iosifovič Averbuch
Lubomír Klapka
Michal Marvan
Kristína Smítalová

External Members:

Miroslav Bartušek (Faculty of Science, Masaryk University, Brno)
Roman Ger (Mathematical Institute, Silesian University, Katowice)
Oldřich Kowalski (Faculty of Mathematics and Physics, Charles University, Prague)
Michal Lenc (Faculty of Science, Masaryk University, Brno)
Josef Mikeš (Faculty of Science, Palacký University, Olomouc)
Štefan Schwabik (Mathematical Institute, Academy of Sciences of the CR, Prague)
Jiří Tolar (Czech Technical University, Prague)

Supervisors for PhD-studies

Geometry and global analysis

Vladimir Iosifovič Averbuch
Miroslav Engliš (Mathematical Institute, Academy of Sciences of the CR, Prague)
Lubomír Klapka
Tomáš Kopf
Demeter Krupka (Faculty of Science, Masaryk University, Brno)
Olga Krupková
Michal Marvan
Jana Musilová (Faculty of Science, Masaryk University, Brno)
Alexandr Vondra

Mathematical Analysis

Vladimir Iosifovič Averbuch
Miroslav Engliš (Mathematical Institute, Academy of Sciences of the CR, Prague)
Štefan Schwabik (Mathematical Institute, Academy of Sciences of the CR, Prague)
Jaroslav Smítal
Kristína Smítalová
Marta Štefánková

Mathematical Physics

Miroslav Engliš (Mathematical Institute, Academy of Sciences of the CR, Prague)
Lubomír Klapka
Tomáš Kopf
Michal Lenc (Faculty of Science, Masaryk University, Brno)

Branch Council for PhD-studies

Academic Council of the Mathematical Institute holds the function of the Branch Council.

Convocations for PhD-studies

Geometry and Global Analysis

Vladimir Iosifovič Averbuch
Miroslav Engliš (Mathematical Institute, Academy of Sciences of the CR, Prague)
Ivan Kolář (Faculty of Science, Masaryk University, Brno)
Michal Marvan
Josef Mikeš (Faculty of Science, Palacký University, Olomouc)
Jaroslav Smítal – chairman

Mathematical Analysis

Vladimir Iosifovič Averbuch
Miroslav Bartušek (Faculty of Science, Masaryk University, Brno)
Ondřej Došlý (Faculty of Science, Masaryk University, Brno)
František Neuman (Academy of Sciences of the Czech Republic, Brno)
Bedřich Půža (Faculty of Science, Masaryk University, Brno)
Štefan Schwabik (Mathematical Institute, Academy of Sciences of the CR, Prague)
Jaroslav Smítal – chairman

Mathematical Physics

Miroslav Engliš (Mathematical Institute, Academy of Sciences of the CR, Prague)
Lubomír Klapka
Michal Lenc (Faculty of Science, Masaryk University, Brno) – chairman
Jiří Tolar (Czech Technical University in Prague)

INFORMATION ON STUDY

The Silesian University in Opava offers programmes and branches of bachelor, master and doctoral study. It is organised in a full-time, extramural and combined form.

Bachelor and master study follow the credit system. Doctoral programme follows an individual study plan made in agreement of the student and his or her supervisor.

Bachelor study, which takes 3 or 4 years, is finished with the final state exam. Graduates will be given the title of Bachelor (baccalareus, Bc.).

Master study takes 5 years (it incorporates bachelor study) and it is also finished with the final state exam. Graduates are given the title of Master of Science (magister, Mgr.). The viva voce of a thesis is a part of the exam.

Graduates who want to be given degree of Doctor of Natural Sciences (Rerum Naturalium Doctor, RNDr.) are firstly required to submit a dissertation. The next step is an oral examination including the viva voce of their dissertation. Having passed this examination, the graduates will be given the mentioned title.

Doctoral study takes 3 years and it is finished with the doctoral state exam. Graduates will be given the title Doctor of Philosophy (PhD.). The viva voce of a PhD-thesis is a part of the exam.

Study Plans in the Mathematical Branches

Mathematical Institute is charged with the realisation of the bachelor master and doctoral studies included in the study programme Mathematics.

Bachelor Study Programme

Study branches:

- Applied Mathematics
- Mathematical Methods in Economics
- Applied Mathematics in Risk Management

Master Study Programme

Study branches:

- Geometry
- Mathematical Analysis
- Mathematical Physics

Doctoral Study Programme

Study branches:

- Mathematical Analysis
- Geometry and Global Analysis
- Mathematical Physics

ORGANISATION OF THE DOCTORAL STUDY FOR FOREIGN STUDENTS

Seminars (Seminar on Differential Geometry and Seminar on Mathematical Analysis) take place in winter as well as in summer semester every academic year.

Study of the other subjects is organised in the individual form through the tuition and it is finished with an exam. Every exam can be realized in English, Czech or Slovak.

Teaching Activities
Doctoral students teach in the bachelor or master courses up to 4 lessons weekly.

Geometry and Global Analysis

Seminar on Differential Geometry and its Application (M. Marvan)

List of Subjects

Algebraic and Differential Topology
Algebraic Structures
Differential Geometry of Manifolds
Functional Analysis and Differential Equations
General Topology
Geometric Methods in General Gelativity and Field Theory
Geometric Methods in Mechanics
Geometric Theory of Differential Equations
Global Analysis
Global Variational Analysis

Mathematical Analysis

Seminar on Mathematical Analysis (J. Smítal)

List of Subjects

Basic Algebraic Categories
Differential Geometry and Its Applications in Mathematical Physics,
Dynamical Systems
Elements of Variational Analysis
Functional Analysis
Mathematical Methods in Natural and Technical Sciences
Ordinary Differential Equations
Theory of Functions
Topology
Variational Analysis

Mathematical Physics[1]

Seminar on Differential Geometry (D. Krupka)
Seminar on Quantum theory (M. Lenc)

List of Subjects

Algebraic and Differential Topology
Geometric Theory of Differential Equations
Global Variational Analysis
Variational Methods in Physics
Topology and Functional Analysis
General Theory of Relativity
Quantum Field Theory
String Theory
Calibration Theory

[1] The study is organized in the co-operation with the Masaryk University in Brno

MASTER'S STUDY PROGRAMME COURSES WHICH COULD BE STUDIED IN ENGLISH

Talented students of the master's studies can join the scientific research within the scientific projects and grants solved in the Mathematical Institute.

Algebraic and Differential Topology I

Year:	IV–V.	Hours weekly:	2/2 Z, Zk*
Semester:	winter	ETCS credits:	6
Lecturer:	Michal Marvan		

Aim and programme of the lectures and seminars

1. Categories, functors, category Top, Gr a Ab; products and sums, pull-back and push-out.
2. Homotopy of continuous mappings, relative homotopy; homotopical equivalence of topological spaces, contractibility.
3. Category Top_h, functors in algebraic topology, elementary problems of algebraic topology, homotopy extension property, Borsuk pairs.
4. Paths and loops, fundamental group, simply-connected spaces.
5. Covering spaces, covering path theorem, covering homotopy theorem, fundamental group, covering mapping theorem
6. Methods of calculation of homotopy groups, G-spaces, fundamental group of the orbit space; Seifert–Van Kampen theorem.
7. Superior homotopic groups, exact sequence of the homotopic groups.

Reference:

C. KOSNIOWSKI, A First Course in Algebraic Topology, Cambridge Univ. Press, Cambridge, 1980.
A. HATCHER, Algebraic Topology, http://www.math.cornell.edu/~hatcher.
S. MAC LANE, Categories for the Working Mathematician, Springer, New York, 1971.

Algebraic and Differential Topology II

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	Michal Marvan		

Aim and programme of the lectures and seminars

1. Complex of abelian groups, homology, morphisms of complexes, algebraic homotopies of morphisms of complexes.
2. Singular simplexes, singular chains, singular homology, homotopic invariance of singular homologies.
3. Long exact sequence of homologies, barycentric subdivision, excision, Mayer–Vietoris formula.

* This notation means: 2 hours of lectures/2 hours of seminars, Z stand for credit, Zk for an exam.

4. Degree of a mapping, methods of calculation.
5. CW complexes, cellular homology, its identification with singular homologies.
6. Homology and cohomology with coefficients; free resolutions, Tor and Ext functors, universal coefficients theorem; Künneth formula, Eilenberg–Zilber theorem.

Reference:

A. HATCHER, Algebraic Topology, http://www.math.cornell.edu/~hatcher.
S. MAC LANE, Homology, Springer, Berlin, 1963.
R.M. SWITZER, Algebraic Topology – Homotopy and Homology, Springer, Berlin, 1975.

Algebraic Structures

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	Michal Marvan		

Aim and programme of the lectures and seminars

1. Algebraic structures and sub-structures, generators, homomorphisms, isomorphisms, congruences, factor algebras, products.
2. Semigroups, monoids, groups, Lagrange theorem, normal subgroups, group action, orbit and stabilizator, Burnside theorem.
3. Rings, fields, ideals.
4. Modules a vector spaces, sums, free modules, tensor product.
5. Lattices.

References:

W.J. GILBERT: Modern Algebra with Applications (Wiley, New York, 1976).
S. LANG: Algebra (Addison-Wesley, Reading, 1965).
S. MAC LANE, G. BIRKHOFF: Algebra (The Macmillan Co., New York, 1967).

Complex Analysis

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Marta Štefánková		

Aim and programme of the lectures and seminars

1. **Mappings and differentiation in the complex plane:** complex plane (representation of the complex numbers, properties), differentiation (definition, analytic function, Cauchy–Riemann equations), conformal mappings (linear mapping, Möbius transformation, exponential functions, power functions, Joukowski function).
2. **Complex integrals:** curve integral in **C** (definition, basic properties), Cauchy integral theorem, independence of integration path, Cauchy integral formula, differentiation of an analytic function, Morera's theorem, Liouville's theorem.
3. **Taylor and Laurent series, singularities:** power series (radius of the convergence, analytic function and its differentiation), Taylor series (Taylor theorem, Taylor series of elementary functions), Laurent series (Laurent theorem), classification of singular points, behaviour of the function in a neighbourhood of singular points.
4. **Integration by the method of residues:** residues (definition, evaluation of residue at poles), Residue theorem, evaluation of real integrals.

5 . **Laplace transformation:** definition, properties (linearity, existence, uniqueness), Laplace transforms of derivatives, shifting on the s-axis and t-axis respectively, $(F(s-a), f(t-a))$.

Reference:

E. KREYSZIG: Advanced Engineering Mathematics, John Wiley and Sons 1979.
R.V. CHURCHIL: Complex variables and Applications, McGraw-Hill Book Company 1960.

Differential Geometry I

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Lubomír Klapka		

Aim and programme of the lectures and seminars

Connection: Decomposition of module of vector fields on fibred manifold to vertical and horizontal part, parallel translation of a vector, holonomy group, linear connection, affine connection, non-linear connection, torsion and curvature, examples.

Geodesics and linear connection: Definitions, star-shaped neighbourhood, convex neighbourhood, exponential mapping, examples.

Riemann and pseudo-Riemann manifolds: Metrics, metric tensor, Levi-Civita connection, Riemann tensor, covariant derivative of tensor fields, Einstein spaces, spaces with constant curvature and homogeneous spaces, examples.

References:

S. KOBAYASHI K. NOMIZU: *Foundations of Differential Geometry*, Interscience, New York 1963
B. O'NEILL: *Semi-Riemannian Geometry*, Academic Press, 1983.
W. KLINGENBERG: *Riemannian geometry*, Studies in Mathematics. 1. Berlin
L.P. EISENHART, *Riemannian geometry,* Princeton–Oxford.

Differential Geometry II

Year:	III.	Hours weekly:	4/2 Z, Zk
Semester:	summer	ETCS credits:	8
Lecturer:	Lubomír Klapka		

Aim and programme of the lectures and seminars

Lie groups a algebras: Definitions and examples, the fifth Hilbert problem ant its solution, Lie algebra tangent to Lie group.

Local theory of Lie groups: Baker–Campbell–Hausdorff formula and its automorphfisms. Coordinate expressions of group operations in left-invariant logarithmic atlas

Differential geometry of Lie groups: Left-invariant and right-invariant vector fields and differential forms. One-parametric Lie subgroups. Exponential and logarithmic mapping. Integrability of left-invariant and right-invariant distributions. Invariant integration.

General linear group: Lie group and algebras of square matrices, their subgroups and subalgebras. Linear, adjoint, true, reducible and completely reducible representations of Lie groups and algebras. Ado theorem.

References:

N. BOURBAKI: *Éléments de mathématique: groupes et algebres de Lie*. Masson, Paris, 1982.

C. CHEVALLEY: *Theory of Lie groups* I (Princeton University Press, Princeton, 1999). 15 editions since 1957

M. POSTNIKOV: *Lectures in Geometry V. Lie Groups and Lie algebras*, Nauka, Moscow, 1982.

Functional Analysis and Optimisation Theory I

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	V.I. Averbuch		

Aim and programme of the lectures and seminars

1. Topological vector spaces. Definitions, examples and basic properties. Properties of neighbourhoods of zero. Theorem on a base of neighbourhoods of zero.
2. Locally convex spaces. Convex sets. Semi-norms. Locally convex topologies generated by a system of semi-norms.
3. Hahn–Banach theorem. Separation theorems.
4. Openness Principle. F-spaces and Fréchet spaces. Banach theorem on inverse mapping. Theorem on closed graph.
5. Boundedness principle. Bounded sets. Bounded operators. Equicontinuity, equiboundedness and pointwise boundedness. Banach–Steinhaus theorem.
6. Duality Theory. Pairing. Dual space. Weak and weakened topologies.
7. Elements of Convex Analysis. Convex and sublinear functions. Minkowski function. Conjugate convex function. Polar. Subdif. Duality theorem. Alaoglu–Bourbaki theorem.
8. Normed Spaces. Definition and examples. Operator norm. Canonical imbedding in the second dual space. Reflexive spaces. Criterion of the weak convergence of sequences. Spectrum of a linear operator. Theorem on the spectrum of a bounded linear operator. Compact operators: definition and basic properties.
9. Hilbert Spaces. Scalar product. Orthogonal projection. Self-duality of Hilbert spaces. Hilbert basis. Orthogonalization procedure. Self-adjoint operators; examples of unbounded self-adjoint operators from quantum mechanics. Hilbert-Schmidt theorem.

References:

N. DUNFORD, J.T. SCHWARTZ: Linear Operators. I. General Theory (Interscience, New York, 1958).

R. EDWARDS: Functional analysis. Theory and applications (Holt, Rinehart and Winston, New York-Toronto-London, 1965).

A.A. KIRILLOV, A.D. GVISHIANI, Theorems and problems of functional analysis, Moscow, 1979 (in Russian).

Functional Analysis and Optimisation Theory II

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	V.I. Averbuch		

Aim and programme of the lectures and seminars

1. Ancient extremal problems.
2. Brachystochrona problem and beginning of the Calculus of Variations.
3. Traffic problem and operational planning.
4. Time optimisation and rise of the optimal control theory.
5. Lagrange multiplicators a Kuhn–Tucker theorem.
6. Variation and Euler equation.
7. Fundamental theorem of the linear programming. Simplex algorithm. Duality.
8. Optimal control problem and Pontrjagin principle.

References:

F.S. HILLIER, G.J. LIEBERMAN, Introduction to Operations Research, Holden Day 1980.

Global Analysis I

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Artur Sergyeyev		

Aim and programme of the lectures and seminars

1. Differentiable manifolds, examples of manifolds, smooth maps between manifolds
2. Submanifolds, immersion and embedding
3. Tangent vectors, tensors and differential forms on manifolds
4. Vector fields on manifolds, flows, Lie derivatives of tensors and differential forms
5. Distributions and the Frobenius theorem
6. Critical points of functions on manifolds, the Sard theorem
7. Partition of unity and the Whitney theorems

References:

P. OLVER: Applications of Lie groups to differential equations, N.Y., Springer,1993.
P. OLVER: Equivalence, invariants and symmetry, Cambridge Univ. Press, 1995.
B.A. DUBROVIN, S.P. NOVIKOV, A.T. FOMENKO: Modern geometry methods and applications, Springer, 1984 (or any other edition).
S. STERNBERG: Lectures on differential geometry, Englewood Cliffs, N.J., Prentice Hall, 1964.
R. NARASIMHAN: Analysis on real and complex manifolds, North-Holland, Amsterdam, 1968.
A. SERGYEYEV: Lectures on global analysis, text for students, MU SU, 2000.
V. AVERBUCH: Global analysis, text for students UT 3/2000, MU SU, 2000.

Global Analysis II

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	Artur Sergyeyev		

Aim and programme of the lectures and seminars

1. Lie groups: definition and properties, the relationship between the Lie algebra and the Lie group, the exponential map, classical Lie groups.

2. Representations of the groups. Transformation groups and their properties.
3. Orientability of manifolds. Integration of differential forms on manifolds: the general Stokes formula. The degree of a mapping and the intersection index.
4. The basics of calculus of variations: jet spaces, total and variational derivatives and contact forms; differential equations as submanifolds in jet space. The action, the Lagrangian and the Euler–Lagrange equations.

References:

J. FUCHS, C. SCHWEIGERT: Lie algebras and representations. Cambridge University Press, Cambridge, 1997.

B.A. DUBROVIN, S.P. NOVIKOV, A.T. FOMENKO: Modern geometry methods and applications, Springer, 1984 (or any other edition).

P. OLVER: Applications of Lie groups to differential equations, N.Y., Springer,1993.

P. OLVER: Equivalence, invariants and symmetry, Cambridge Univ. Press, 1995.

S. STERNBERG: Lectures on differential geometry, Englewood Cliffs, N.J., Prentice Hall, 1964.

R. NARASIMHAN: Analysis on real and complex manifolds, North-Holland, Amsterdam, 1968.

F. WARNER: Foundations of differentiable manifolds and Lie groups. Scott, Foresman and Co., Glenview, Ill.-London, 1971 (or any later edition).

A. SERGYEYEV: Lectures on global analysis, text for students, MU SU, 2000.

Numerical Analysis

Year:	IV–V.	Hours weekly:	4/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	Karel Hasík		

Aim and programme of the lectures and seminars

1. Errors (method errors, round-off errors). Significant digits. Absolute and relative errors. Correct problems, ill-conditioned systems and stable algorithms.
2. Approximation theory (philosophy of the curve-fitting). Least squares method. Normal equations.
3. Polynomial approximation. Orthogonalization of approximating polynomials.
4. Splines.
5. Interpolation. Lagrange polynomial. Error of Lagrange interpolation. Newton polynomial.
6. Difference calculus. Fraser diagram, Newton forward-difference formula.
7. Solution of linear equations. Determinants, Gaussian elimination, direct factorisation of matrices (LU-decomposition).
8. Iterative techniques in matrix algebra (Jacobi and Gauss–Seidel iterative method). Convergence of a method.
9. Numerical solutions of nonlinear equations. General one-step and multi-step method. Newton–Raphson method, the bisection method, the method of false position.
10. Roots of polynomials, Sturm sequence.
11. Numerical integration. Numerical quadrature, trapezoidal rule, Simpson’s rule. Errors of the methods.
12. Initial-value problems for ordinary differential equations. Solution in the power-series form and Picard approximations.

13. Euler's method. Runge–Kutta method. Order of a method.
14. The shooting method for boundary value problems of ordinary differential equations.
15. Finite-difference method for boundary value problems of partial differential equations.

References:

R.L. BURDEN, J.D. FAIRES: Numerical Analysis. PWS-Kent, Boston, 1985.

A. RALSTON, P. RABINOWITZ: A first course in numerical analysis. Dover Publications, Inc., Mineola, NY, 2001 (reprint of the 1978)

Ordinary Differential Equations

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Lubomír Klapka		

Aim and programme of the lectures and seminars

1. **Differential equations:** basic definitions and procedures, ordinary (ODE) and partial (PDE) differential equations, putting ODE into the standard form $y' = f(x, y)$.

2. **ODEs in the normal form:** General case: existence of a solution, local and global uniqueness of the solution, integral form of the solutions, manifold of the maximal solutions. Autonomous case: phase space, trajectories and their properties, local group of transformations, classification of the solutions. Linear case: domain of the maximal solution, resolution and its properties, homogeneous and non-homogeneous case.

Linear autonomous case: domain of the maximal solutions, group of transformations, matrix exponential.

Stability. Lyapunov stability, asymptotical stability, stability criteria, Lyapunov function.

3. **ODEs of the second order in one dependent variable:** Properties of solutions. Sturm theorem, oscillations, boundary problems.

References

W.T. REID: *Ordinary differential equations*, John Wiley, New York 1971.

L.S. PONTRJAGIN, *Obyknovennyje differenciaľnyje uravnenija*, Nauka, Moskva 1965 (in Russian). English translation: Addison-Wesley, London–Paris 1962.

L. SCHWARTZ, *Analyse mathématique II*, Hermann, Paris 1967 (in French); *Analiz*, Tom. II, Mir, Moskva 1972 (in Russian).

Partial Differential Equations I

Year:	III.	Hours weekly:	2/2 Z,
Semester:	summer	ETCS credits:	6
Lecturer:	Jana Kopfová		

Aim and programme of the lectures and seminars

1. Basic notations and definitions. Some known equations. Well posed problems. Generalised solutions. Short history of PDEs.
2. PDE's of first order. Cauchy problem. Characteristic ordinary differential equations. Homogenised linear equations of first order. Quasilinear equations. Nonlinear equations of first order. Plane elements. Monge cone.
3. Cauchy initial problem. Cauchy–Kowalewska theorem. Generalised Cauchy problem. Characteristics.

4. Classification of equations of second order. Linear PDE's with constant coefficients. Linear PDE's of second order: reduction to the canonical form.
5. Parabolic equations. Derivation of the physical model. Correctly stated boundary value problems. Cauchy problem: fundamental solution; existence and uniqueness theorem. Maximum principle.
6. Fourier method. Boundary value problems for parabolic equations. Hyperbolic equations. The Laplace equation on a circle.
7. Hyperbolic equations. Method of characteristics. D'Alembert formula. Hyperbolic equations on a halfline and on a finite interval. Three-dimensional wave equation. Riemann method for the Cauchy problem. Riemann formula.
8. Elliptic equations. Laplace equation. Poisson equation. Physical motivation. Harmonic functions. Symmetric solutions. Maximum principle. Uniqueness of solutions.

Partial Differential Equations II

Year:	III.	Hours weekly:	2/2 Z,
Semester:	winter	ETCS credits:	6
Lecturer:	Jana Kopfová		

Aim and programme of the lectures and seminars

1. Elliptic equations. Potentials: volume potential, simple layer potential, double layer potential. Green formulas. Generalised Green formula. Harmonic functions: Dirichlet integral, Gauss integral theorem. Dirichlet problem and Neumann problem. Poisson formula.
2. Elements of distribution theory. Test functions. Decomposition of the unity. Localization. Support. Regular and singular distributions.
3. Operations over distributions. Convolution.
4. Method of integral transforms. The Fourier transform. The Laplace transform.
5. Modern methods of solving PDEs. Sobolev spaces. Generalised solutions. Lax–Milgram theorem.

References:

M. RENARDY, R.C. ROGERS: *An introduction to partial differential equations*, Springer, New York, 1993.
V. AVERBUCH: *Partial differential equations*, Silesian University, Opava 2000.
L.C. EVANS: *Partial differential equations*, Amer. Math. Soc., Providence, 1998.
L. HÖRMANDER, *The analysis of linear partial differential operators* I–IV, Grundlehren der Mathematischen Wissenschaften 265, 257, 274, 275 (Springer, Berlin, 1983–1985).

Real Analysis I

Year:	III.	Hours weekly:	2/0 Z,
Semester:	winter	ETCS credits:	4
Lecturer:	Petra Šindelářová		

Aim and programme of the lectures and seminars

I. **Measures**. Basic properties of measures. Outer measures and Carathéodory theorem. Extension theorem of measures. Measures on metric space. Hausdorff measures. Lebesgue–Stieltjes Measures and Lebesgue Measures.

II. **Measurable Functions**. Measurable functions. Approximations by simple measurable functions. Sequences of measurable functions.

References

See Real analysis II

Real Analysis II

Year: III.
Semester: summer
Lecturer: Petra Šindelářová
Hours weekly: 2/0 Z, Zk
ETCS credits: 4

Aim and programme of the lectures and seminars

III. **Integration.** Integrals of simple non-negative functions. Lebesgue–Stieltjes integral and Lebesgue integral. Relations between Riemann and Lebesgue integral. Mean-Value theorems.

IV. **Differentiation**. Dini derivatives. Continuity and differentiation. Differentiation of monotone functions. Points of discontinuity. Darboux property. The Banach–Mazurkiewicz theorem. Functions of bounded variation. Absolutely continuous functions.

References

A.M. BRUCKNER, J.B. BRUCKNER, B.S. THOMSON, *Real Analysis*, Prentice-Hall, Upper Saddle River, NJ, 1997.

Seminar on Real Analysis I

Year: III.
Semester: winter
Lecturer: Petra Šindelářová
Hours weekly: 0/2 Z,
ETCS credits: 4

Aim and programme of the lectures and seminars

Exercises on Measures and Measurable Functions.
Solving problems from the journal *American Mathematical Monthly*.

Seminar on Real Analysis II

Year: III.
Semester: summer
Lecturer: Petra Šindelářová
Hours weekly: 0/2 Z,
ETCS credits: 4

Aim and programme of the lectures and seminars

Exercises on Integration and Differentiation
Solving problems from the journal *The American Mathematical Monthly*.

References

A.M. BRUCKNER, J.B. BRUCKNER, B.S. THOMSON, *Real Analysis*, Prentice-Hall, Upper Saddle River, New Jersey 07458, 1997.

American Mathematical Monthly, An Official Publication of the Mathematical Association of America.

Theory of Probability and Statistics

Year:	II.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Martin Snethlage		

Aim and programme of the lectures and seminars

1. **Discrete sample space:** basic definitions, examples, uniform distribution, combinatorics, hypergeometric distribution, random variables.
2. **Conditional distribution and independence:** conditional probability, Bayes formula, independent events, independent random variables.
3. **Moments:** expected value conditional expectation, variance, covariance, correlation.
4. **Statistics:** point estimation, hypothesis testing, confidence intervals.
5. **Continuous random variables:** basic definitions, examples, Gaussian distribution, statistic for Gaussian distributed random variables.

References:

W. FELLER, An Introduction to Probability Theory and Its Applications. Vol. 1., New York, J. Wiley & Sons, 1968.
D. FREEDMAN et al.: Statistics. New York, W. W. Norton & Comp., 1991.

Topology

Year:	III.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Olga Krupková		

Aim and programme of the lectures and seminars

1. Topological structure on a set (open and closed set, exterior, interior, frontier, basis of a topology).
2. Continuous mapping, homeomorphism.
3. Construction of a topological space (subspace, products, factor space).
4. Metric spaces (metrics, metric topology, complete metric space, equicontinuous mapping, contraction, fixed point theorem, isometry, Hausdorff theorem).
5. Compact and locally compact topological spaces.
6. Convergence in a topological space (in first type of countability space, in metric space).
7. Connected and arcwise-connected topological space.
8. Regular, normal and paracompact spaces, topological manifolds.

References:

J.L. KELLEY, *General Topology* (Van Nostrand, Princeton, 1955).
J.R. MUNKRES: Topology, A First Course (Prentice Hall, New Jersey 1975).

Variational Analysis I

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	winter	ETCS credits:	6
Lecturer:	Artur Sergyeyev		

Aim and programme of the lectures and seminars

1. Geometrical foundations: jet spaces, total derivatives and contact forms; differential equations as submanifolds in jet space.
2. Vector fields on jet spaces. Prolongations. Point, contact and generalised symmetries. Recursion operators. The action. The Lagrangian and its variation. The variational derivative. The Euler–Lagrange equations. The relationship between the invariance of action and
3. the invariance of the Euler–Lagrange equations. Variational symmetries.
4. The conservation laws. Triviality of conservation laws and Lagrangians. The relationship between symmetries and conservation laws: the first Noether theorem.
5. The infinite-dimensional symmetry groups. Gauge transformations. The second Noether theorem.

References:

See Variational analysis II.

Variational Analysis II

Year:	IV–V.	Hours weekly:	2/2 Z, Zk
Semester:	summer	ETCS credits:	6
Lecturer:	Artur Sergyeyev		

Aim and programme of the lectures and seminars

1. Poisson structures. Hamiltonian systems and their integrals The notion of complete integrability and the Liouville theorem. Reduction of Hamiltonian systems and the momentum map.
2. Bihamiltonian systems and their properties. The Hamilton–Jacobi equation. Separation of variables in Hamiltonian systems.
3. The relationship between Lagrangian and Hamiltonian systems. The Legendre transformation. Regularity. The Hamiltonian version of the first Noether theorem.
4. The variational bicomplex and the inverse problem of calculus of variations. Hamiltonian partial differential equations.

References (Variational Analysis I and II):

V. ARNOLD: Mathematical methods of classical mechanics. Springer, New York, 1999 (or any other edition).

I. DORFMAN: Dirac structures and integrability of nonlinear evolution equations. Wiley & Sons, Chichester, 1993.

A.T. FOMENKO: Symplectic geometry. Gordon and Breach, New York, 1988.

I.M. GELFAND, S.V. FOMIN: Calculus of variations. Prentice-Hall, Englewood Cliffs, 1963 (or any later edition).

M. GIAQUINTA, S. HILDEBRANDT: Calculus of variations, Vol. I–II. Springer, Berlin, 1996.

O. KRUPKOVÁ: The geometry of ordinary variational equations. Springer, Berlin, 1997.

P.J. OLVER: Applications of Lie groups to differential equations. Springer, New York, 1993 (or any later edition).

OTHER COURSES

After an agreement with the director of the Mathematical Institute, it is possible to attend the following courses, (they also could be available in English)

ETCS	Subject	Hours weekly	semester	exam
6	Algebraic and Differential Topology III	2/2	w	Z, Zk
6	Algebraic and Differential Topology IV	2/2	s	Z, Zk
6	Cathegory Theory	2/2	w	Z, Zk
6	Chapters from Topology I	2/2	w	Z, Zk
6	Chapters from Topology II	2/2	s	Z, Zk
6	Chapters from Functional Analysis I	2/2	w	Z, Zk
6	Chapters from Functional Analysis II	2/2	s	Z, Zk
6	Differential Invariants	2/2	w	Z, Zk
6	Dynamical Systems I	2/2	w	Z, Zk
6	Dynamical Systems II	2/2	s	Z, Zk
6	Introduction to the Theory of Lie Groups	2/2	s	Z, Zk
6	Geometric Methods in Physics I	2/2	w	Z, Zk
6	Geometric Methods in Physics II	2/2	s	Z, Zk
6	Geometric Theory of Partial Differential Equations I	2/2	w	Z, Zk
6	Geometric Theory of Partial Differential Equations II	2/2	s	Z, Zk
6	Mathematical Foundations of the General Theory of Relativity I	2/2	w	Z, Zk
6	Mathematical Foundations of the General Theory of Relativity II	2/2	s	Z, Zk
6	Logic and Theory of Sets	2/2	s	Z, Zk
6	Numerical Methods in the Theory of Relativity I	2/2	s	Z, Zk
6	Numerical Methods in the Theory of Relativity II	2/2	w	Z, Zk
6	Variational Analysis on Manifolds	2/2	s	Z, Zk

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