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A MODEL OF STOCK PRICES BEHAVIOR

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Abstract. The considered model describes dynamics of intrinsic value and actual price of stocks on the stock markets. The model construction is based on characterization market participants behavior and on relationships between stock market and economical environment. Our model respects different behavior of market participants at undervalued and overvalued market. Our aim is to contribute to study of processes, which influence the price dynamics at the stock markets.

1. Introduction

In the financial groups there are known some models, describing behavior of the market prices on the financial markets [1], [8]. The models respect supply and demand low, which determines a market stock price. These ones insist on characterization of market participants behavior and their purpose is to create results, which can be compared with actual market data. By comparing these data we can measure the quality of model approximation.

A conception of modelling introduced in this paper is based on the reciprocal relation between market price and intrinsic value. The market price is determined by supply and demand low. The intrinsic value is the classic quantity of fundamental analysis established by B. Graham, D. L. Dodd and J. B. Williams in 1930's. The intrinsic value represents theoretically calculated quantity, which indicates, what should be the true market stock price. By comparing of the market stock price and the intrinsic value we distinguish undervalued and overvalued stocks. The undervalued stock has lower market price than its intrinsic value. The overvalued stock has higher market price than its intrinsic value. Market participants use different computational models for assignment of the intrinsic value. The most used models are dividend discount models, incremental models, balancing models and financial analysis.

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As an example of often used computational model it is possible to mention the dividend discount model:

$$F = \frac{d_1}{(1+r)} + \frac{d_2}{(1+r)^2} + ... + \frac{d_n}{(1+r)^n},$$

where the parameters have the following sense:

F	...	stock intrinsic value ,
$d_1 - d_n$	...	prospective dividents in particular years,
r	...	bank rate derived from rates of interest (in form of decimal number).

Intrinsic value according to this model is given by sum of actual values of future dividends. But generally, an intrinsic value is considerably subjective quantity. Investors use own methods for its calculation in dependence on their capital strategy and they establish own expected values. Variability of supply and demand volume in the market depends on activity of market participants, who give impulse to buy or sale. Market participants behavior is determined by effort to gain maximal capital appreciation and by their acceptance of the risk. From this point of view we can define two basic patterns of market participants behavior, rational and speculative.

Rational market participants ("racionalists") are less tolerant to the risk and they press for deuce between market price and intrinsic value of stock.

Speculative market participants ("speculators") are more tolerant to the risk and they press for the maximum difference between market price and intrinsic value, their object is to obtain the highest benefit from this difference.

Market participants can approach from one pattern of behavior to the other in relation to market situation and own preferences. We can suppose, that dynamics of market price is determined by intensity of rational, respectively speculative behavior of market participants and by immediate difference between the market price and the intrinsic value. Next we suppose, that dynamics of the intrinsic value depends on macro and microeconomics conditions and on influence of stock market to evolution of this quantity.

2. Model of stock prices behavior

We introduce following notation:

P......market price of stock,

F......intrinsic value of stock,

k_1......"racionalists" behavior intensity coefficient ,

k_2......"speculators" behavior intensity coefficient ,

k_3......coefficient expressed the influence of macro and microeconomics conditions on dynamics of intrinsic value of stock,
k_4......coefficient expressed the influence of stock market on dynamics of intrinsic value.

Both market price P and intrinsic value F appear in the model as indices. A base of index calculation is intrinsic value. Coefficients k_1 and k_2 can be determined by count of rational and speculative market participants and by their dealing activity. Coefficient k_3 represents expected growth of the stock company and it can be deduced from complex analysis of the stock. Coefficient k_4 represents sensibility of reaction of the stock company and business environment to price information accorded by stock market. At the model construction we suppose, that the coefficients are constants, but in the real situation we can expect their distinct variability at time. Coefficients k_i, $i = 1, ..., 4$ represent non-dimensional numbers. We suppose that $k_{1,2,4} \geq 0$ and $k_3 \in R$. Concerning the state variebles, we consider the situation, when

$$P > 0 \quad \text{and} \quad F \geq 0. \tag{2.1}$$

2.1. **The case of undervalued market.** We can consider the undervalued market as the situation when $P < F$. The model is represented by the system of differential equations

$$\begin{aligned} \dot{P} &= \frac{k_1}{P^2}(F - P) - k_2 P, \\ \dot{F} &= k_3 F + k_4 (P - F) F. \end{aligned} \tag{2.2}$$

The first equation describes the dynamics of market price and it consists of two components. The first one expresses the influence of the "racionalists" $\frac{k_1}{P^2}(F - P)$ and the second one the influence of the "speculators" $k_2 P$. "Rationalists" aspire return the stock market prices to their intrinsic value. Speed of return depends on depth of undervaluation. It is expressed by linearly difference $F - P$ and progressively by component P^2 in denominator. "Speculators" struggle against effort of "rationalists" and they deepen undervaluation of market price. Their effort with grow of undervaluation falls.

The second equation describes dynamics of intrinsic value and it consists of components of influence of macro and microeconomics conditions $k_3 F$ and influence of stock market on dynamics of intrinsic value $k_4(P - F)F$.

System of differential equations (2.2) has the unique stationary point satisfying condition (2.1). It is

$$[\overline{P},\overline{F}] \quad = \quad \left[\sqrt[3]{\frac{k_1k_3}{k_2k_4}}, \frac{k_3}{k_4}+\sqrt[3]{\frac{k_1k_3}{k_2k_4}}\right], \tag{2.3}$$

which is defined for

$$k_i > 0, \quad i = 1, \ldots, 4. \tag{2.4}$$

Let us denote $\alpha = \frac{k_1k_3}{k_2k_4}$. After linearization of the system (2.2), we count of Jacobi matrix of the system (2.2) in the point (2.3) and we get the characteristic equation

$$\begin{aligned} &\lambda^2 + (k_3 + k_4\sqrt[3]{\alpha} + \frac{2k_1k_3}{k_4\alpha} + k_1\sqrt[3]{\frac{1}{\alpha^2}} + k_2)\lambda \\ &+\frac{2k_1k_3^2}{k_4\alpha} + 3k_1k_3\sqrt[3]{\frac{1}{\alpha^2}} + k_1k_4\sqrt[3]{\frac{1}{\alpha}} + k_2k_4\sqrt[3]{\alpha} + k_2k_3 \quad = \quad 0. \end{aligned} \tag{2.5}$$

The sign of real parts of eigenvalues of equation (2.5) we determine by using

$$\lambda^2 + k\lambda + q = (\lambda-\lambda_1)(\lambda-\lambda_2) = \lambda^2 + (-\lambda_1-\lambda_2)\lambda + \lambda_1\lambda_2. \tag{2.6}$$

If condition (2.4) is satisfied then it holds in equation (2.5), that $q > 0$ and $k > 0$. Real parts $Re\lambda_{1,2}$ of characteristic equation roots (2.5) are negative numbers. Stationary point (2.3) is locally asymptotically stable.

2.2. **The case of overvalued market.** We can consider overvalued market as a situation, when $P > F$. Model is represented by the system of the differential equations

$$\begin{aligned} \dot{P} &= -k_1(P-F)P^2 + \frac{k_2}{P}, \\ \dot{F} &= k_3F + k_4(P-F)F. \end{aligned} \tag{2.7}$$

An economic background of system (2.7) is similar as in the case undervalued market. "Rationalists" depress overvaluation and "speculators" increase it.

The differential system (2.7) has two stationary points satisfying condition (2.1)

$$[\overline{P_1}, \overline{F_1}] = \left[\sqrt[4]{\frac{k_2}{k_1}}, 0\right] \quad k_i > 0 \quad i = 1, 2, \tag{2.8}$$

$$[\overline{P_2}, \overline{F_2}] = \left[\sqrt[3]{-\frac{k_2 k_4}{k_1 k_3}}, \frac{k_3}{k_4} + \sqrt[3]{-\frac{k_2 k_4}{k_1 k_3}}\right] \tag{2.9}$$

where $k_i > 0, \quad i = 1, 2, 4$ and $k_3 < 0$.

Let us denote $\beta = -\frac{k_2 k_4}{k_1 k_3}$. After linearization of system (2.7), we count of Jacobi matrix of the system (2.7) in the points (2.8) and (2.9) and we get the characteristic equations

$$\lambda^2 - (k_3 + k_4 \sqrt[4]{\frac{k_2}{k_1}} - 4\sqrt{k_1 k_2})\lambda - 4\sqrt{k_1 k_2}(k_3 + k_4 \sqrt[4]{\frac{k_2}{k_1}}) = 0, \tag{2.10}$$

$$\begin{aligned} &\lambda^2 + (k_3 + k_4 \sqrt[3]{\beta} + k_1 \sqrt[3]{\beta^2} - \frac{2k_1 k_3}{k_4} \sqrt[3]{\beta} + k_2 \sqrt[3]{\frac{1}{\beta^2}})\lambda \\ &-2k_1 k_3 \sqrt[3]{\beta^2} - \frac{2k_1 k_3^2}{k_4} \sqrt[3]{\beta} + k_2 k_3 \sqrt[3]{\frac{1}{\beta^2}} + k_2 k_4 \sqrt[3]{\frac{1}{\beta}} = 0. \end{aligned} \tag{2.11}$$

From analysis of characteristic equation (2.10) it follows, that $Re\lambda_1 > 0$ and $Re\lambda_2 < 0$ iff $-\frac{k_3}{k_4} < \sqrt[4]{\frac{k_2}{k_1}}$. The stationary point (2.8) is unstable and it is a saddle point. In the analysis of characteristic equation (2.11) we use the relation (2.6). Real parts of the roots of characteristic equation (2.11) are negative iff $q > 0$ and $k > 0$. A simple calculation yields

$$\begin{aligned} q &> 0 \quad \text{iff} \quad k_1 k_3^4 < k_2 k_4^4, \\ k &> 0 \quad \text{iff} \quad 27 k_1^2 k_2 > k_3 k_4^2. \end{aligned} \tag{2.12}$$

If conditions (2.12) are satisfied then stationary point (2.9) is locally asymptotically stable.

3. Limitations of models and suggestion of their solution

Both models have two problems when they are compared with market reality. The first one is the fact, that models do not exhibite oscillatory behavior, which is typical for real evolution of market stock price. The second problem is the limit between undervalued and overvalued market with $P = F$. So, the models do not describe continuous transition from undervalued to overvalued market. Approximately both problems can be removed when the view of "speculators" activity is more specified. We will still expect, that "speculators" press for deepening of market undervaluation or overvaluation, but only at the situation, when the price history supports this trend. This new view at "speculators" behavior corresponds with chartists behavior mentioned in [1] and it corresponds to technical analysis of stocks.

After instalation the above mentioned principles we can create one model for all the market situation at the following form:

$$\begin{aligned} \dot{P} &= k_1(F-P) - k_2(P_h - P), \\ \dot{F} &= k_3 F + k_4(P-F)F \end{aligned} \tag{3.1}$$

where $P_h = P(t-h)$.

We used system of differential equations (2.2) for undervalued market. At the first equation there was removed progressive part P^{-2} from rational component and to speculative component there was established delay of market price about time horizon h. The second equation of system (2.2) remains unchanged.

4. Orientation of another effort

For definition of initial problem of system (3.1) we must find function of historic market price P_h. This function we can obtain from the real market price time-series. It is obvious, that various sizes of time horizon h affords the various functions P_h. Next we can suppose, that ability of prediction of the market price will be differentiated for various time horizons h and historic functions P_h. So we will find congruous horizon h, which allows repeatedly the best approach between the model and real date.

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