X

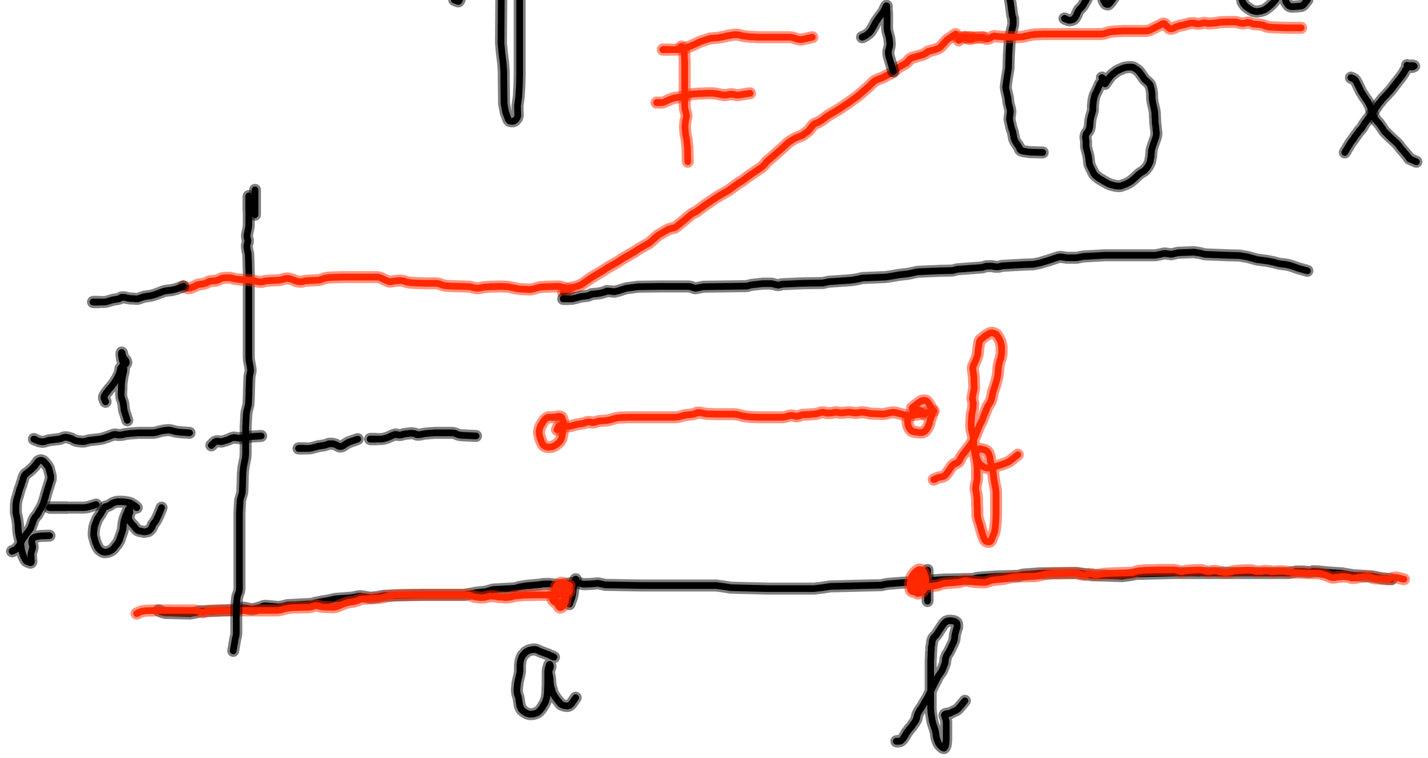
$$P(a \le X < b) = \int_a^b f(x)\,dx$$

$$P(a < X \le b)$$

$$\vdots$$

Def. X má <u>rovnoměrné</u> rozdělení na (a, b)

$$f(x) = \begin{cases} 0 & x \leq a \\ \frac{1}{b-a} & x \in (a, b) \\ 0 & x \geq b \end{cases}$$



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

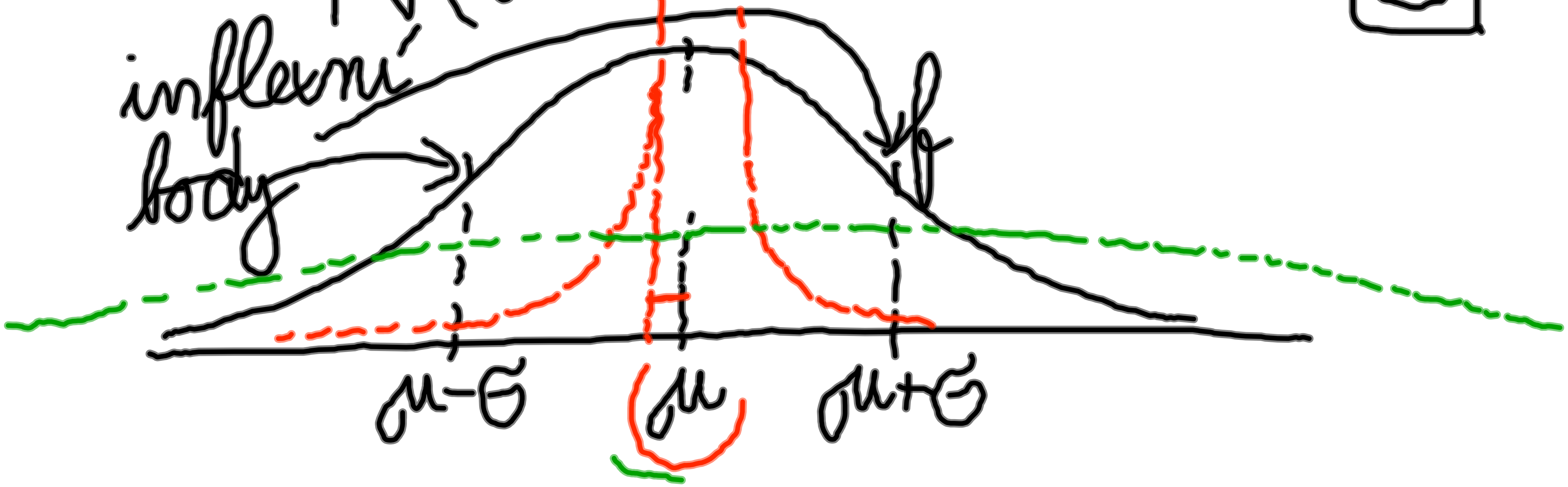
Def. X má **normální rozdělení** s parametry $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^+$ pokud pro hustotu platí

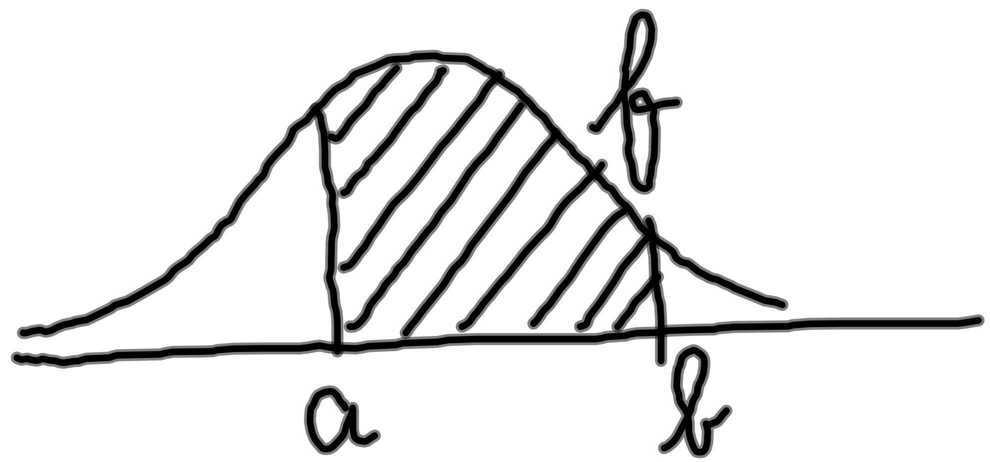
$$f(x) = \frac{1}{\sigma\boxed{\sqrt{2\pi}}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\boxed{X \sim N(\mu, \sigma)} \qquad N(5, \underbrace{3}_{\boxed{\sigma}})$$

$$N(\mu, \sigma^2)$$





μ

σ

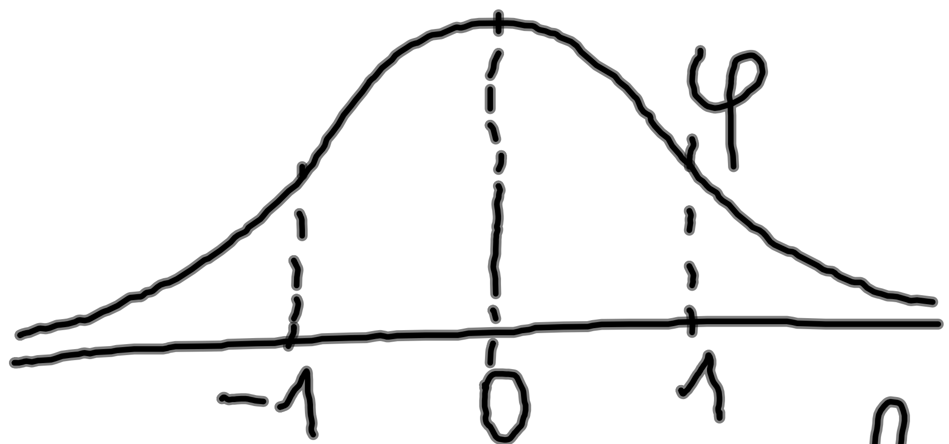
$\mu = 0$

$\sigma = 1$

normované normální

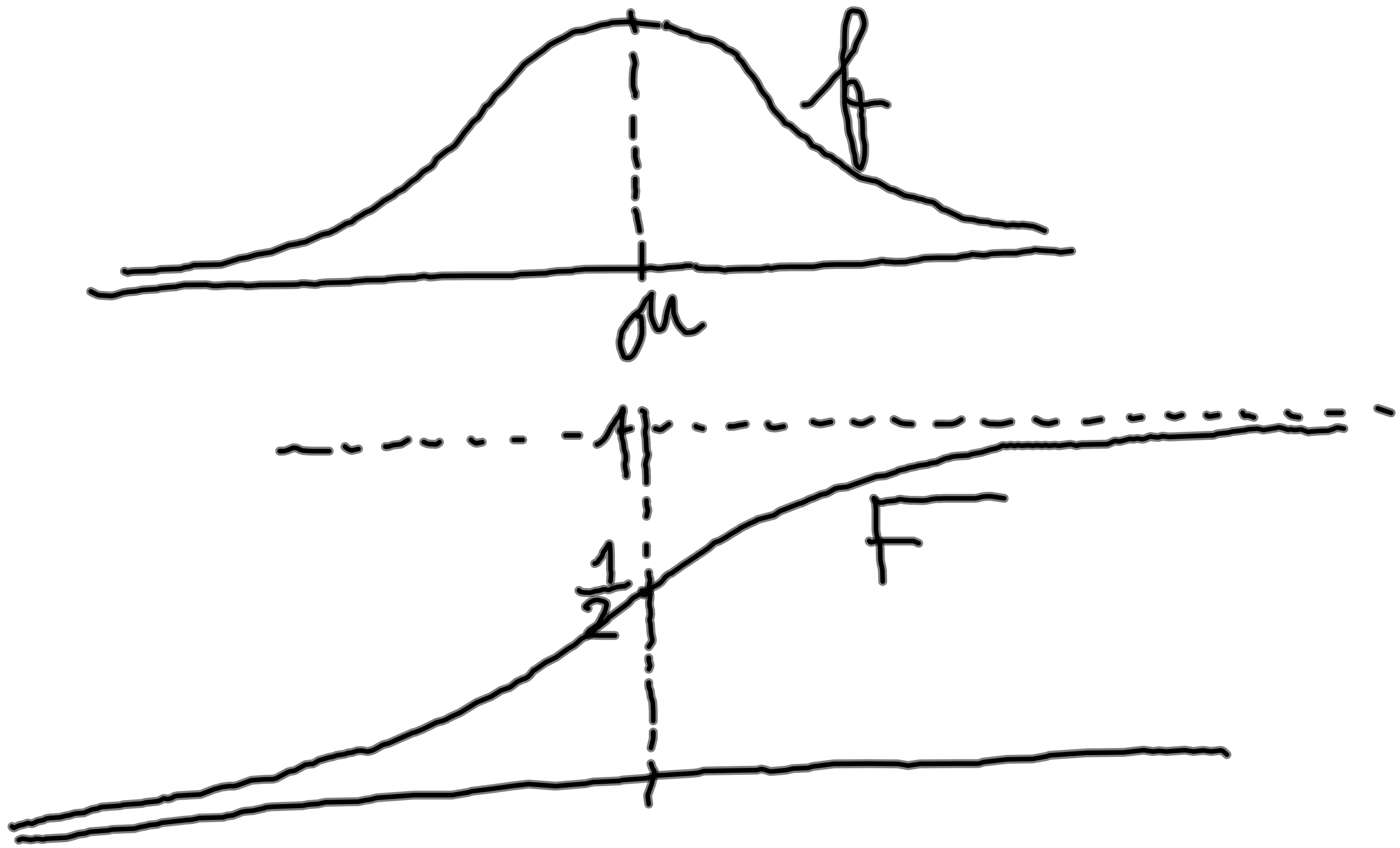
$N(0,1)$

hustota $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$



distribuční funkce

$$\boxed{\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}}\, du}$$ tabelované


f
μ
1
1/2
F

Tvrzení: $X \sim N(\mu, \sigma)$, $F \dots$ d.f.

Platí: $\boxed{F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)}$

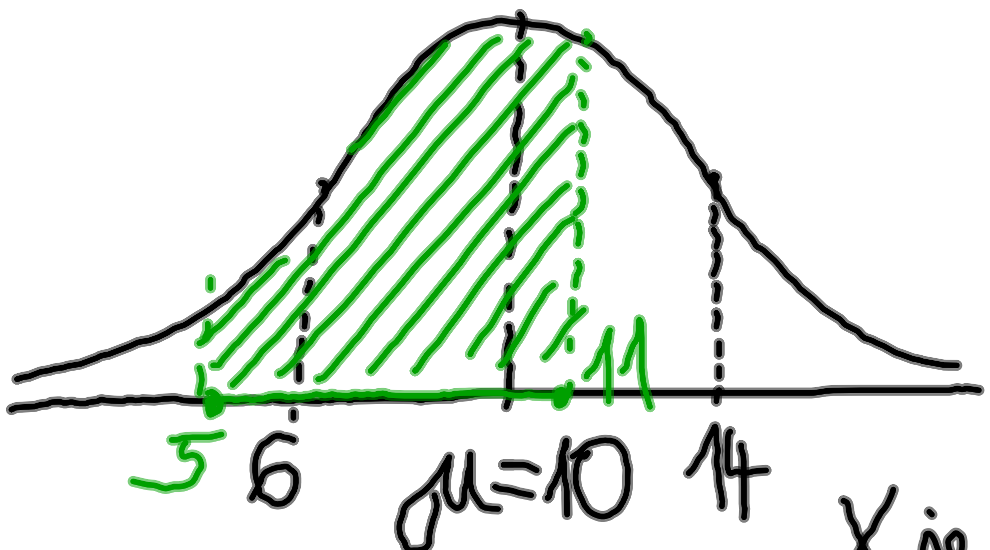
Důkaz:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(s-\mu)^2}{2\sigma^2}}\, ds = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{u^2}{2}}\, du$$

$$\frac{s-\mu}{\sigma} = u$$

Př. $X \sim N(10,4)$ $\quad \mu = 10$, $\sigma = 4$

P_1 že $X \in [5,11]$



$P(5 \le X \le 11) =$

$= P(5 \le X < 11) =$

X je spojitá

$= F(11) - F(5) = \Phi\left(\frac{11-10}{4}\right) - \Phi\left(\frac{5-10}{4}\right) = \Phi(0{,}25) - \Phi(-1{,}25)$

Číselné charakteristiky

Střední hodnota $E(X)$

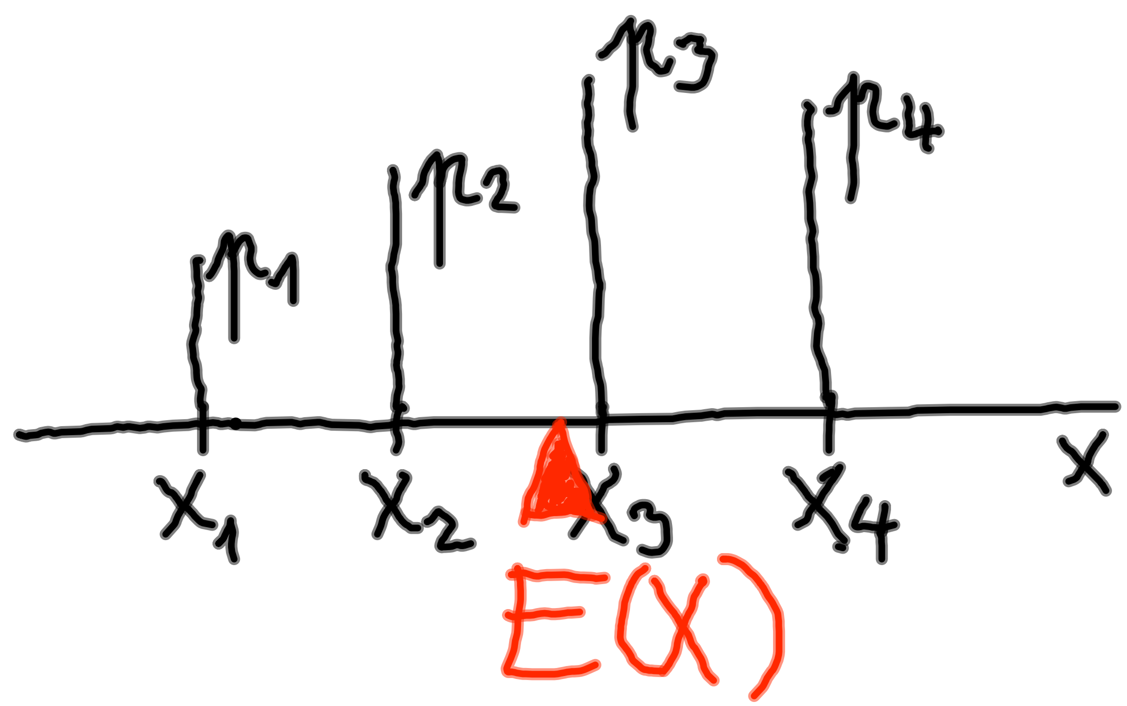
X diskrétní

$x_1, x_2, \ldots$

$P(X = x_i) = p_i, \quad i = 1, 2, \ldots$

$$\boxed{E(X) = \sum_i x_i p_i}$$

X



charakteristika polohy

$$E(Y) = E(X) + a$$

Pozn. 2

X

$$Y = X + a$$

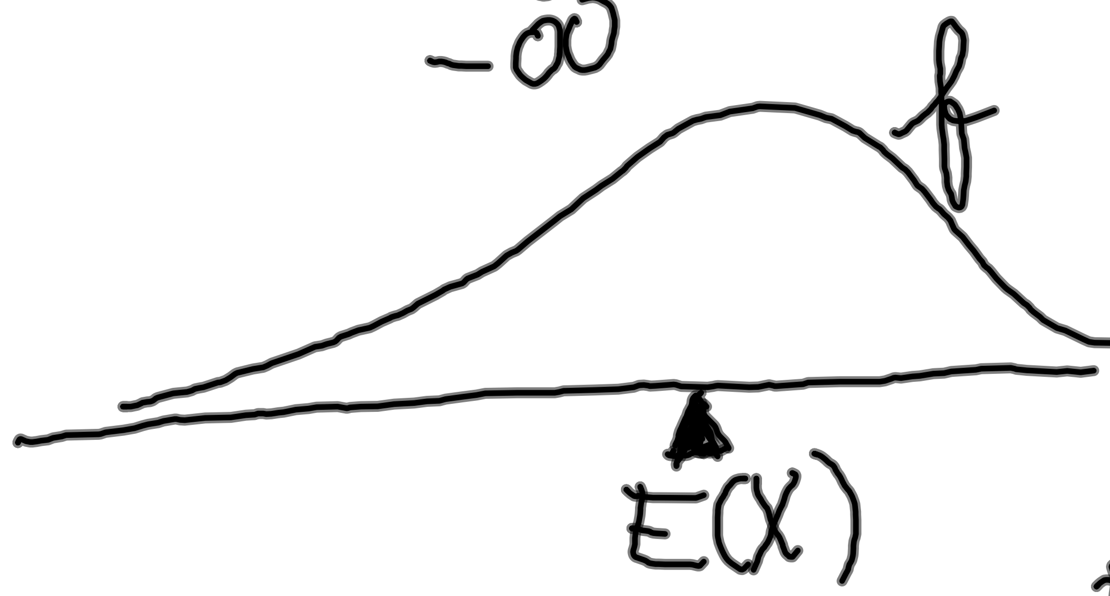
$$y_i = x_i + a$$

$$P(Y = y_i) = P(X = x_i) = p_i$$

$$E(Y) = \sum_{i=1}^{n} y_i p_i = \sum_{i=1}^{n} (x_i + a) p_i = \underbrace{\sum_{i=1}^{n} x_i p_i}_{E(X)} + a \overbrace{\sum_{i=1}^{n} p_i}^{1}$$

X je <u>spojitá</u> s hustotou f

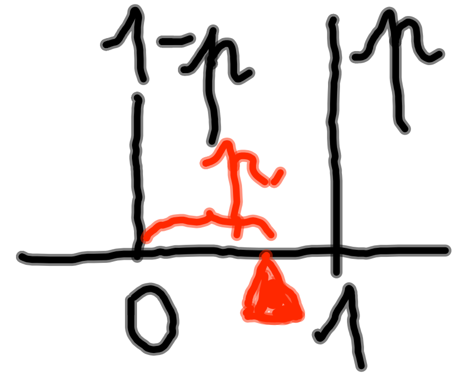
$$E(X)=\int_{-\infty}^{\infty} x\, f(x)\, dx$$



$X+a$

$E(X+a)=E(X)+a$

Př. X má alternativní rozdělení



$E(X) = ?$

$E(X) = 0 \cdot (1-p) + 1 \cdot p = p$

<u>Pr.</u> $X \sim Bi(n, p)$ $P(A) = p$

$k = 0, 1, 2, \ldots, n$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E(X) = ?$$

$$E(X) = \sum_{k=0}^{n} \underbrace{k\binom{n}{k}} p^{k}(1-p)^{n-k}$$

$$k\binom{n}{k} = k\,\frac{n!}{k!\,(n-k)!} = \frac{\overbrace{n!}^{n\,(n-1)!}}{(k-1)!\,\underbrace{(n-k)!}_{(n-1-(k-1))!}}$$

$$= n\binom{n-1}{k-1}$$

$$E(X) = \sum_{k=0}^{n} \underbrace{k\binom{n}{k}}_{n\binom{n-1}{k-1}} p^k (1-p)^{n-k} =$$

$$= n \sum_{k=1}^{n} \binom{n-1}{k-1} p^k (1-p)^{n-k} =$$

$$= n \sum_{k=0}^{n-1} \binom{n-1}{k} \underbrace{p^{k+1}}_{p\,p^k} (1-p)^{n-k-1}$$

$$= n\,p \underbrace{\sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{(n-1)-k}}_{=1} \qquad (\underbrace{p + (1-p)}_{1})^{n-1}$$

$$= \underline{\underline{n\,p}}$$

Př. $X \sim N(\mu, \sigma)$

$$E(X) = \underline{\mu}$$