(X,Y)

kovariance

$$\mathrm{cov}(X,Y)=E\left[(X-E(X))(Y-E(Y))\right]$$

$$\mathrm{cov}(X,X)=$$

$$\mathrm{cov}(X,Y)=\underbrace{E(XY)}-\underbrace{E(X)}\underbrace{E(Y)}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xy\,\underbrace{f(x,y)}\,dx\,dy$$

koeficient korelace X a Y

$$\rho(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sqrt{D(X)D(Y)}}$$

míra lineární závislosti X a Y

$$\boxed{Y = aX + b}\ , \quad a, b \in \mathbb{R},\ a \neq 0$$

$$D(Y) = a^2 D(X)$$

$$\operatorname{cov}(X,Y) =$$

$$= E\Big[(X - E(X))\big(aX + b - \underbrace{E(aX+b)}_{aE(X)+b}\big)\Big]$$

$$= E\Big[a\underbrace{(X - E(X))^2}\Big] = \underline{a\,D(X)}$$

$$\rho(X,Y) = \frac{a\,D(X)}{\sqrt{D(X)\,a^2 D(X)}} = \frac{a\,\cancel{D(X)}}{\underbrace{\sqrt{a^2}}_{|a|}\sqrt{\cancel{D(X)}^2}} = \begin{cases} 1 & a > 0 \\ -1 & a < 0 \end{cases}$$

$$\underbrace{-1}\leq \varrho(X,Y)\leq \underbrace{1}$$

$$\varrho(X,Y)=0$$

$$\operatorname{cov}(X,Y)=0$$

X,Y nekorelované

Nezávislé náhodné proměnné

(X, Y)

A, B nezávislé

$$\boxed{P(A \cap B) = P(A)P(B)}$$

<u>Def.</u> X, Y jsou nezávislé

pokud

$$\underbrace{F(x,y)} = \underbrace{F_1(x)}_{\text{distribuční funkce n.p. } X} \underbrace{F_2(y)}$$

$$\underbrace{P(x_1 \le X < x_2, y_1 \le Y < y_2)}_{\text{primik jevi}} =$$

$$= F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) + F(x_1, y_1)$$

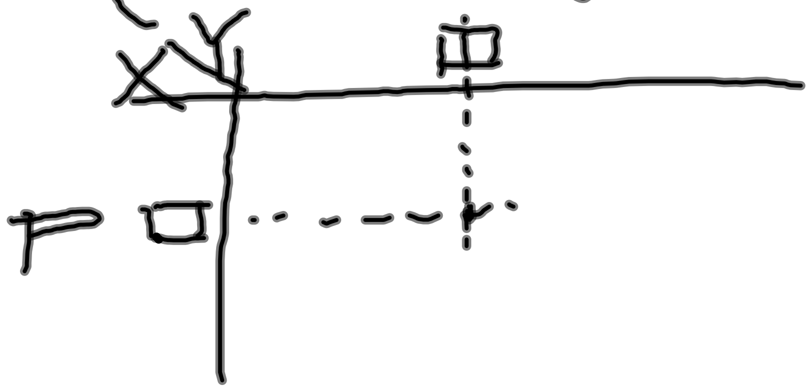
$$= F_1(x_2)F_2(y_2) - F_1(x_2)F_2(y_1) - F_1(x_1)F_2(y_2) + F_1(x_1)F_2(y_1) = F_1(x_2)[F_2(y_2) - F_2(y_1)]$$

$$- F_1(x_1)[F_2(y_2) - F_2(y_1)] =$$

$$= \underbrace{[F_1(x_2) - F_1(x_1)]}_{P(x_1 \le X < x_2)}\underbrace{[F_2(y_2) - F_2(y_1)]}_{P(y_1 \le Y < y_2)}$$

(X, Y) náhodný vektor diskrétneho typu

$$P(X=x, Y=y) = P(X=x)P(Y=y)$$



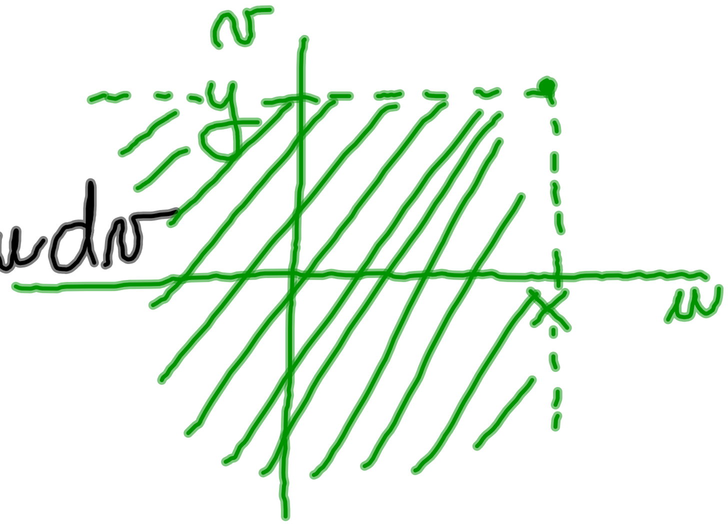
(X,Y) je spojitého typu

$$\boxed{f(x,y) = f_1(x)\, f_2(y)}$$

$$F(x,y) = \underbrace{F_1(x)}\,F_2(y) =$$

$$= \int_{-\infty}^{x} f_1(u)\,du \int_{-\infty}^{y} f_2(v)\,dv =$$

$$= \int_{-\infty}^{x}\int_{-\infty}^{y} \underbrace{f_1(u)\,f_2(v)}_{f(u,v)}\,du\,dv$$



X, Y nezávislé

$$\boxed{E(XY) = E(X)E(Y)} = E(Y)\underbrace{\int_{-\infty}^{\infty} x f_1(x)\,dx}_{E(X)}$$

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_1(x) f_2(y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} x f_1(x)\, y\, f_2(y)\,dy\right)dx =$$

$$= \int_{-\infty}^{\infty} x f_1(x) \underbrace{\left(\int_{-\infty}^{\infty} y f_2(y)\,dy\right)}_{E(Y)} dx =$$

X, Y <u>nezávislé</u>

$$\boxed{D(X+Y) = D(X) + D(Y)}$$

$$D(X+Y) = E\left[\left(X+Y-\underbrace{E(X+Y)}_{E(X)+E(Y)}\right)^2\right]$$

$$= E\left[\left(\left[X-E(X)\right]+\left[Y-E(Y)\right]\right)^2\right]$$

$$= \underbrace{E\left[\left(X-E(X)\right)^2\right]} + E\left[\left(Y-E(Y)\right)^2\right]$$

$$+ 2E\left[\left(X-E(X)\right)\left(Y-E(Y)\right)\right]$$

$$= D(X) + D(Y) + 2\,\mathrm{cov}(X,Y) = 0$$

$$\mathrm{cov}(X,Y) = \underbrace{E(XY)}_{E(X)E(Y)} - E(X)E(Y)$$

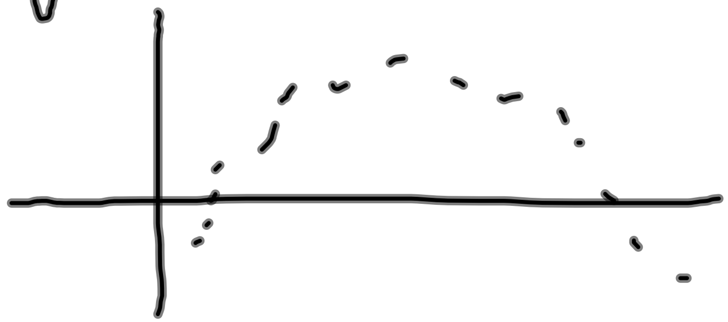
$$= D(X) + D(Y)$$

X, Y nezávislé

$\Longrightarrow \operatorname{cov}(X,Y)=0$

X, Y jsou nekorelované

opak neplatí



Funkce více náhodných proměnných

X^2

$$\mathbb{X}_n = (X_1, \dots, X_n)$$

$$f(x_1, \dots, x_n)$$

$$Y = h(\underline{X}_1, \underline{\dots}, \underline{X}_n)$$

$$X_1^2 + X_2^2 + \dots + X_n^2$$

$\mathbb{X} = (X_1, X_2)$ $X_1 + X_2$

$f(x_1, x_2)$

$Y = \underbrace{h(X_1, X_2)}$

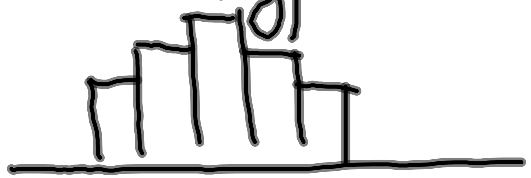
F… distribuční funkce n.p. Y

$$F(y) = \iint\limits_{h(x_1,x_2)<y} f(x_1, x_2)\,dx_1\,dx_2$$

$< y$

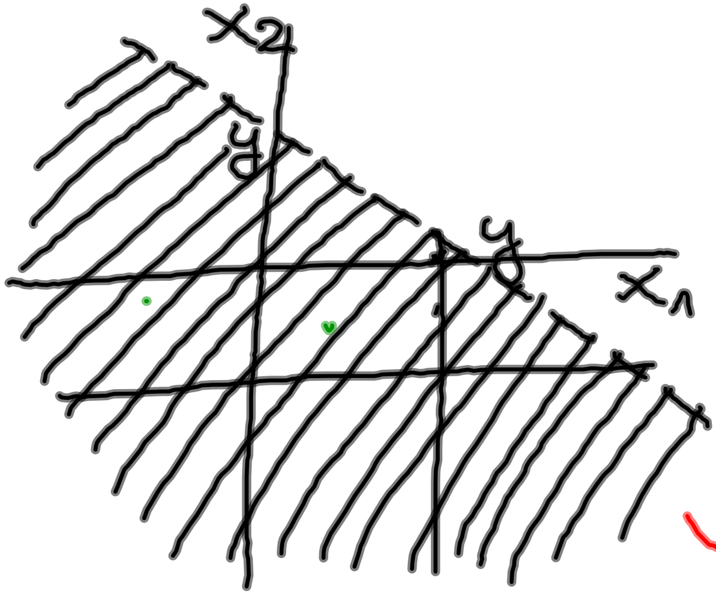
Př. $\mathbb{X} = (X_1, X_2)$ spojitého typu

$Y = X_1 + X_2$



$$F(y) = \iint\limits_{x_1+x_2<y} f(x_1,x_2)\,dx_1\,dx_2 =$$

$x_2 < y - x_1$



$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{y-x_1} f(x_1,x_2)\,dx_2\right)dx_1$$

$$= \boxed{\int_{-\infty}^{\infty}\left(\int_{-\infty}^{y-x_2} f(x_1,x_2)\,dx_1\right)dx_2}$$

X_1, X_2 nezávislé

$$f(x_1,x_2) = f_1(x_1)\,f_2(x_2)$$

$$F(y) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{y-x_2} f_1(x_1)\,f_2(x_2)\,dx_1\right)dx_2$$

$$= \int_{-\infty}^{\infty} f_2(x_2)\underbrace{\left(\int_{-\infty}^{y-x_2} f_1(x_1)\,dx_1\right)}_{F_1(y-x_2)}dx_2 = \int_{-\infty}^{\infty} F_1(y-x_2)\,f_2(x_2)\,dx_2$$

$$f(y) = \underbrace{\int_{-\infty}^{\infty} \underline{f_1}(y - x_2)\,\underline{f_2}(x_2)\,dx_2}_{f_1 * f_2(y)}$$

konvoluce