PS121010.pdf

$$(\Omega, \mathcal{F}, P)$$

1. $\Omega = \{\omega_1, \dots, \omega_N\}$

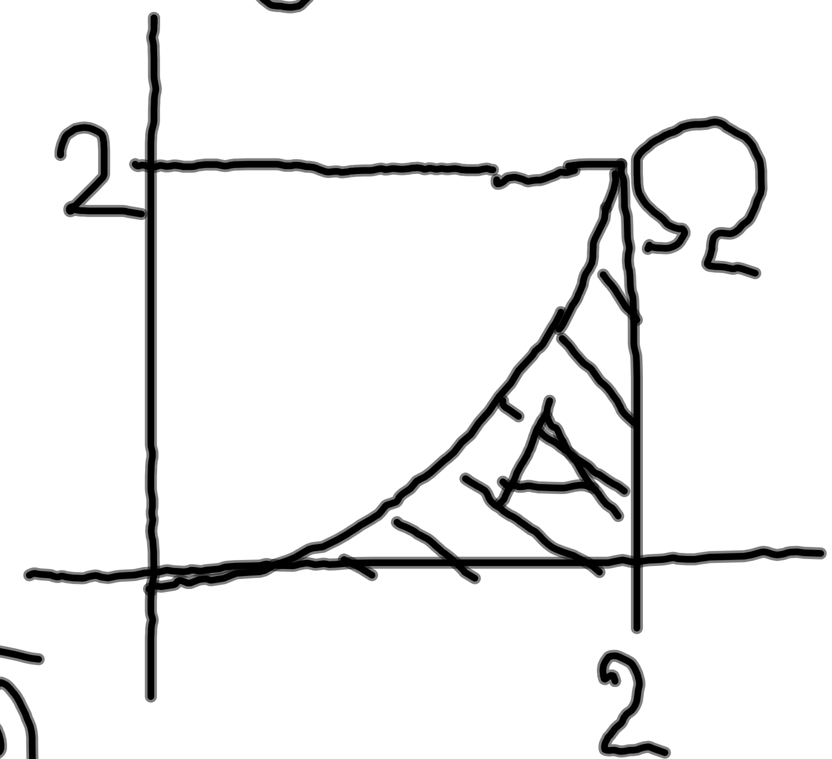
$\mathcal{F} = \mathcal{P}(\Omega)$

$A \in \mathcal{F} \qquad P(A) = \frac{N_A}{N}$

2. $\Omega = [0,2] \times [0,2]$

$A = \left\{ (x,y) \in \Omega \mid 0 \le y \le \frac{x^2}{2} \right\}$

$\mathcal{F} = \{ \emptyset, \Omega, A, \bar{A} \}$



$B \in \mathcal{F}$

$P(B) = \frac{m(B)}{m(\Omega)}$

$$m(A) = \int_0^2 \frac{x^2}{2}\,dx = \frac{4}{3}$$

$$P(A) = \frac{\frac{4}{3}}{4} = \frac{1}{3}$$

geometrická definice pravděpodobnosti

Vlastnosti pravděpodobnosti:

(i) konečná aditivita

$A_1, A_2, \dots, A_n \in \mathcal{S}$, navzájem disjunktní

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

Dikaz:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P\left(\bigcup_{i=1}^{\infty} B_i\right) =$$

$$B_1 = A_1 \qquad =$$

$$\vdots$$

$$B_n = A_n$$

$$B_{n+1} = \emptyset$$

$$\vdots$$

(ii) $P(\bar{A}) = 1 - P(A)$

$$P(\underbrace{A \cup \bar{A}}_{\Omega}) = P(A) + P(\bar{A})$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

důkaz: $A \cup B = \underbrace{B} \cup \underbrace{(A \setminus B)}$

$$P(A \cup B) = P(B) + P(A) - P(A \cap B)$$

$$P(A\setminus B) = P(A) - P(A\cap B)$$

$$A = \underbrace{(A\cap B)}\cup\underbrace{(A\cap \overline{B})}_{A\setminus B}$$

$$P(A) = P(A\cap B) + P(A\setminus B)$$

(iv) $A, B \in \mathcal{S}$

$$A \subset B \Rightarrow P(A) \leq P(B)$$

D. $A \subset B \Rightarrow B = \underbrace{A}_{} \cup \underbrace{(B \setminus A)}_{}$

$$P(B) = P(A) + \underbrace{P(B \setminus A)}_{\geq 0}$$

(v) $A \subset B$

$$\Rightarrow P(B \setminus A) = P(B) - P(A)$$

Náhodná proměnná

$\Omega = \mathbb{R}$

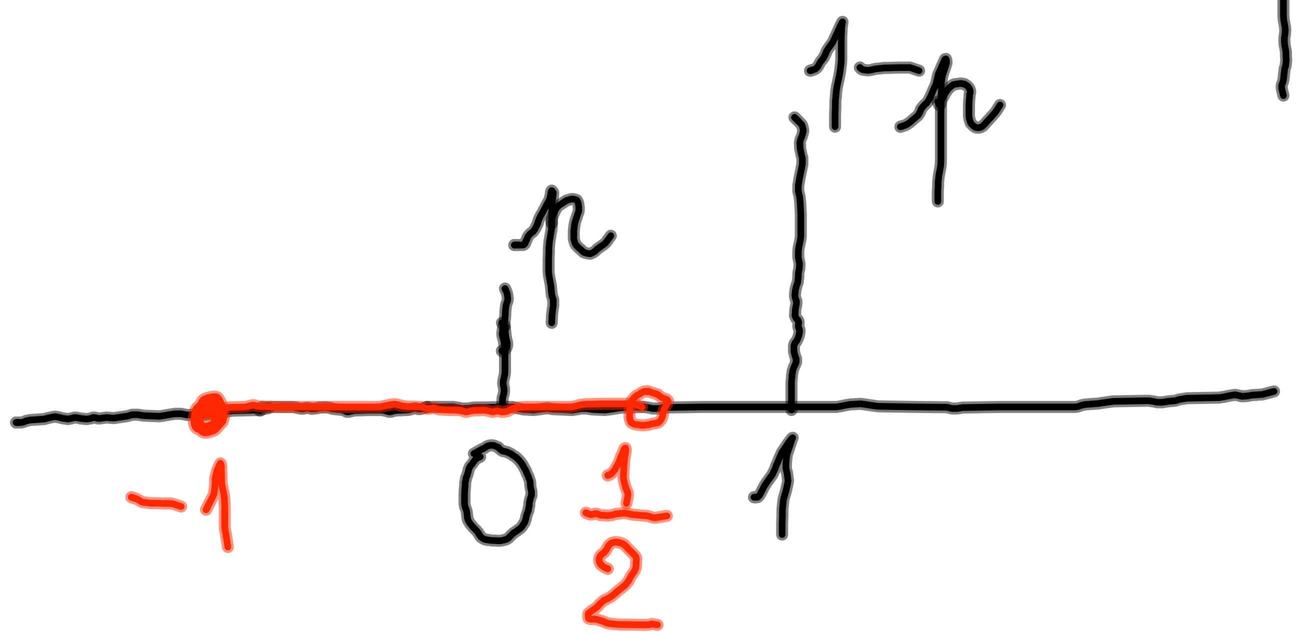
$\Omega \subset \mathbb{R}$

vyberu dobrý výrobek 0

vadný 1

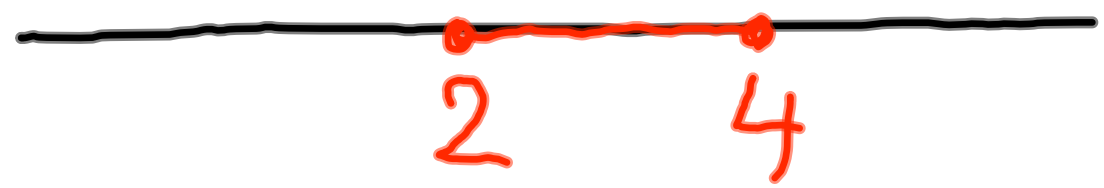
X, Y

$X : \Omega \longrightarrow \mathbb{R}$



$P([-1, \frac{1}{2})) = p$

X



Pouze konečný (spočetný) součet kladných čísel může být konečný.

$$P(X \in [a, b)), \quad a, b \in \mathbb{R}, \quad a < b$$

$$P(\{\omega \in \Omega \mid X(\omega) \in [a, b)\})$$

musí být $\{\omega \in \Omega \mid X(\omega) \in [a, b)\} \in \mathcal{S}$

$$\forall x \in \mathbb{R}: \quad \underline{\{\omega \in \Omega \mid X(\omega) < x\} \in \mathcal{F}}$$

$$\{\omega \in \Omega \mid X(\omega) \in [a, b)\} =$$

$$= \underbrace{\{\omega \in \Omega \mid X(\omega) < b\}}_{\in \mathcal{F}} \setminus \underbrace{\{\omega \in \Omega \mid X(\omega) < a\}}_{\mathcal{F}}$$

<u>Def.</u> $(\Omega, \mathcal{S}, P)$ pravděpodobnostní prostor

Náhodná proměnná

$$X : \Omega \longrightarrow \mathbb{R}$$

$$\forall x \in \mathbb{R} : \ \{\omega \in \Omega \mid X(\omega) < x\} \in \mathcal{S}$$

$$\Omega = \mathbb{R}$$

$$\forall x \in \mathbb{R}: \quad \underbrace{(-\infty, x) \in \mathcal{S}}$$

$$[x, \infty)$$

Def. Distribuční fce

n. p. X

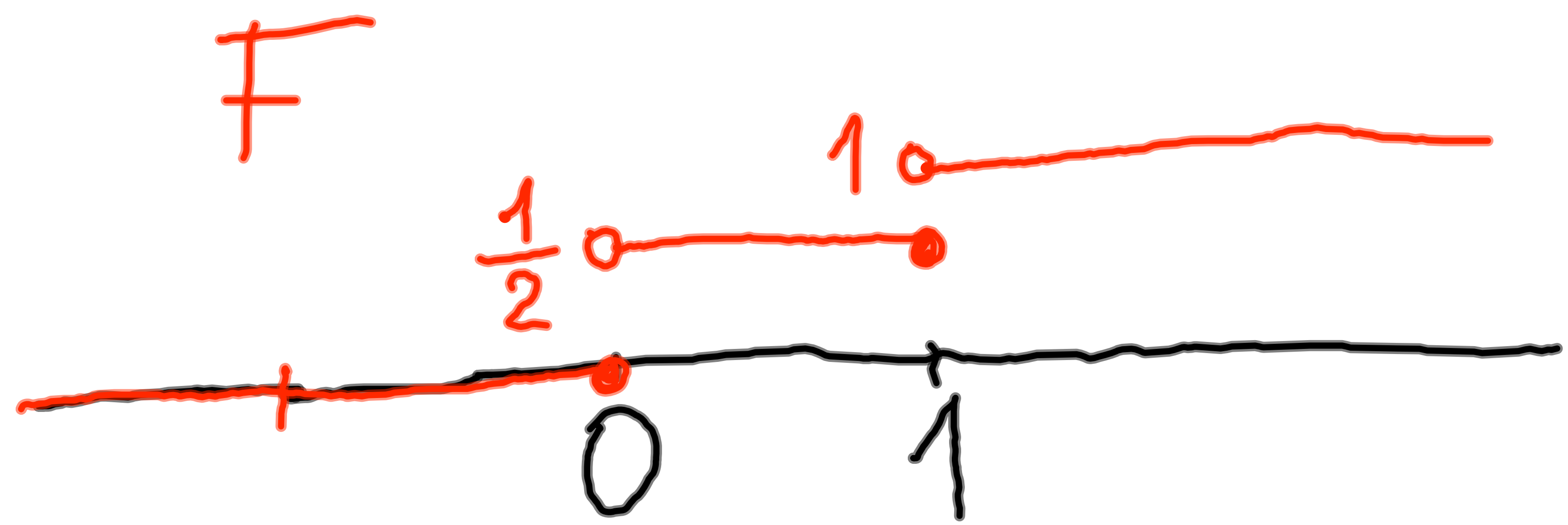
je $F: \mathbb{R} \to \mathbb{R}$

$\forall x \in \mathbb{R}: F(x) = P(\{\omega \in \Omega \mid X(\omega) < x\})$

$F(x) = P(X < x)$

Př. hod mincí, X: líc $\mapsto 0$
rub $\mapsto 1$

$$P(X=0)=P(X=1)=\frac{1}{2}$$



<u>Př.</u> hod kostkou, $\Omega = \{1,2,3,4,5,6\}$

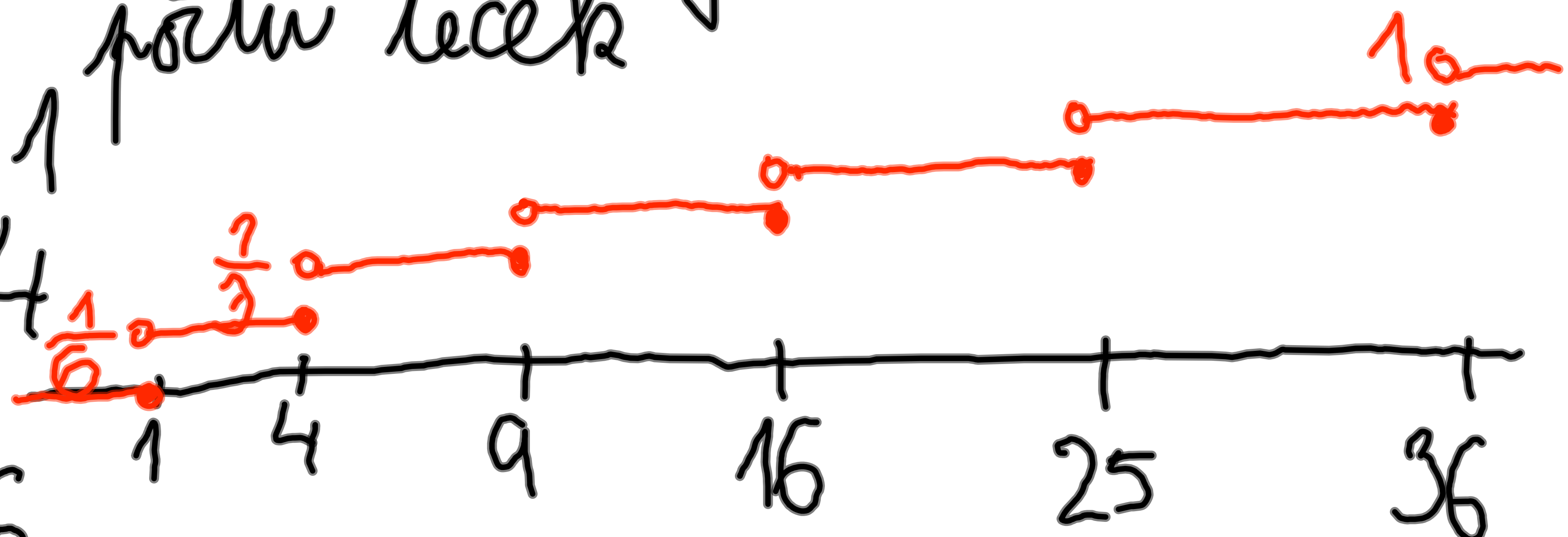
X ... přiřadí druhé mocniny počtu teček $\qquad p_i = \frac{1}{6}$

$X(1) = 1$

$X(2) = 4$

$\vdots$

$X(6) = 36$



Vlastnosti distribuční funkce:

(i) $\forall a, b \in \mathbb{R},\ a < b$:

$$P(\underbrace{a \le X < b}_{X \in [a,b)}) = F(b) - F(a)$$

(ii) neklesající

(iii) zleva spojitá $\forall x_0 \in \mathbb{R}$

(iv) $\lim\limits_{x \to \infty} F(x) = 1$

(v) $\lim\limits_{x \to -\infty} F(x) = 0$

$0 \leq F(x) \leq 1$