Intervalové odhady

$X \sim N(\mu, \sigma)$

$$(X_1, \dots, X_n)$$

$$\frac{\overline{X}-\mu}{\sigma}\sqrt{n} \sim N(0,1)$$

$$X \sim N(\mu, \sigma) \Longrightarrow U = \frac{X-\mu}{\sigma} \sim N(0,1)$$

s distribuční funkcí F

distr. funkce G

$$G(u) = P(\underbrace{U<u}_{\underbrace{\frac{X-\mu}{\sigma}<u}_{X<\mu+\sigma u}}) = P(X<\mu+\sigma u)$$

$$= F(\mu+\sigma u)$$

$$g(u) = \frac{dG}{du} = \sigma f(\mu + \sigma u) = $$

$$= \cancel{\sigma} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{(\cancel{\mu}+\sigma u-\cancel{\mu})^2}{2\sigma^2}}$$

$$\boxed{= \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}}$$

hustota $N(0,1)$

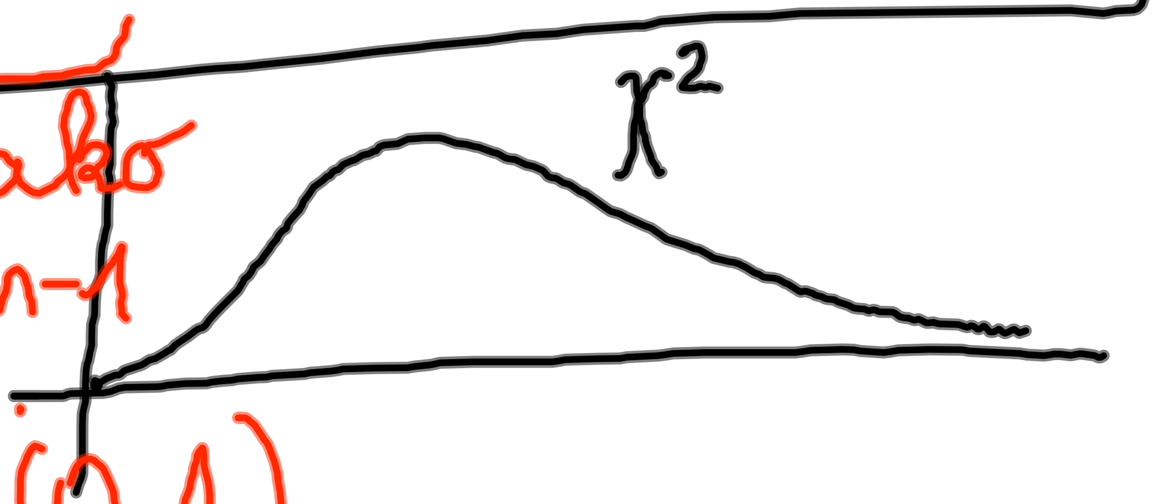
$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{m}}\right)$$

$$\underbrace{\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{m}}}} \sim \underline{N(0,1)}$$

Platí:

$$\boxed{\underbrace{\frac{(n-1)S^2}{\sigma^2}} \sim \chi^2(n-1)}$$

lze vyjádřit jako součet čtverců n−1 nezávislých n. p. s rozdělením N(0,1)



$X \sim N(0,1)$

$Y \sim \chi^2(n)$

$$\frac{X}{\sqrt{\frac{Y}{n}}} \sim t(n)$$

Platí:

$$\frac{\overline{X}-\mu}{S}\sqrt{n} \sim t(n-1)$$

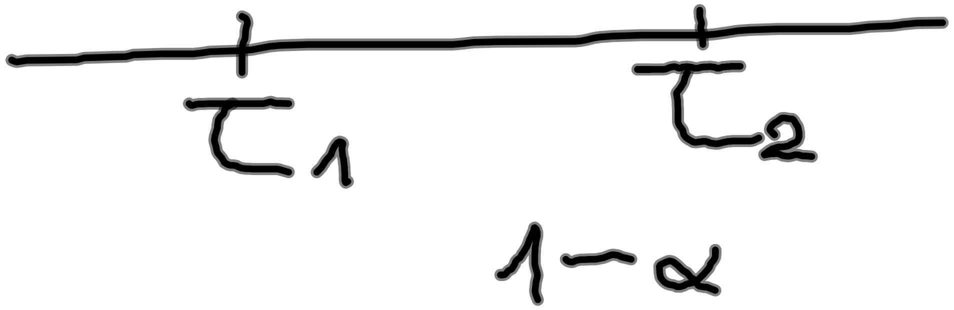
Proč?

$$\frac{\overbrace{(\overline{X}-\mu)\sqrt{n}}}{\sigma\sqrt{\frac{(n-1)S^2}{\sigma^2(n-1)}}} \quad \begin{matrix} N(0,1) \\ \chi^2(n-1) \end{matrix}$$

Intervalový odhad střední hodnoty μ normálního rozdělení

$$X \sim N(\underset{?}{\underline{\boxed{\mu}}}, \underset{?}{\underline{\sigma}})$$

$$(X_1, \ldots, X_n)$$



$$(\tau_1, \tau_2) = I_{1-\alpha}$$

α: $\alpha = 0{,}05$, $\alpha = 0{,}01$

$\underline{I_{1-\alpha}}$

$\underline{(1-\alpha)100}$ – procentní interval spolehlivosti (konfidenční interval)

$\underline{\tau_1}, \underline{\tau_2}$

$\alpha 100\ \%$

σ neznáme

$-t_{1-\frac{\alpha}{2}}(n-1) = t_{\frac{\alpha}{2}}(n-1) \qquad t_{1-\frac{\alpha}{2}}(n-1)$

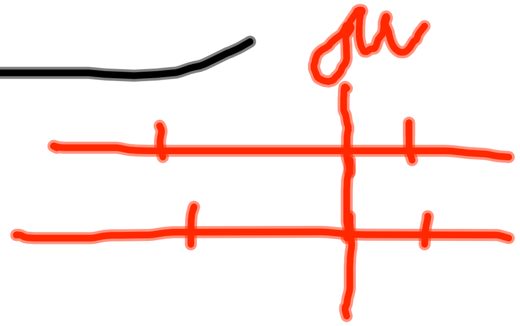
$(1-\frac{\alpha}{2})$-kvantil

$$\frac{\bar{X}-\mu}{S}\sqrt{n} \sim t(n-1)$$

α zvolím

$$P\Big(\underbrace{-t_{1-\frac{\alpha}{2}}(n-1) < \frac{\bar{X}-\mu}{S}\sqrt{n} < t_{1-\frac{\alpha}{2}}(n-1)}\Big) = 1-\alpha$$

$$\cdot\ P\left(\underbrace{-t_{1-\frac{\alpha}{2}}(n-1) < \frac{\bar{X}-\mu}{S}\sqrt{n}}\ \underbrace{< t_{1-\frac{\alpha}{2}}(n-1)}\right) = 1-\alpha$$



$$\underbrace{\mu < \bar{X} + \frac{S t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}}}$$

$$\wedge\ \underbrace{\bar{X} - \frac{S t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}} < \mu}$$

$$P\left(\underbrace{\bar{X} - \frac{S t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}}} < \mu < \underbrace{\bar{X} + \frac{S t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}}}\right) = 1-\alpha$$

$$I_{1-\alpha} = \left(\overline{X} - \frac{S\, t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}}, \overline{X} + \frac{S\, t_{1-\frac{\alpha}{2}}(n-1)}{\sqrt{n}} \right)$$

$(x_1, \dots, x_n)$ realizace náhodného výběru

$\longrightarrow \overline{x}, S$

z tabulek $\underline{t_{1-\frac{\alpha}{2}}(n-1)}$

$n = 19$

$\alpha = 0{,}05$

$t_{1-\frac{\overset{\alpha}{0{,}05}}{2}}(18) = 2{,}1009$

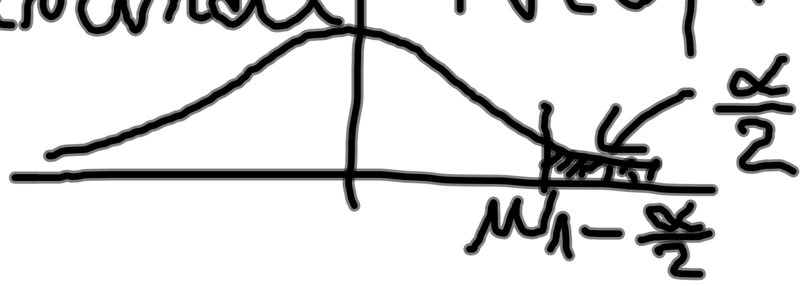
$t_{1-\frac{\alpha}{2}}(n-1)$

$\underline{\sigma}$ známe

$$\underline{\frac{\overline{X}-\mu}{\sigma}\sqrt{n} \sim N(0,1)}$$

$$I_{1-\alpha} = \left(\overline{X} - \frac{\sigma u_{1-\frac{\alpha}{2}}}{\sqrt{n}}, \overline{X} + \frac{\sigma u_{1-\frac{\alpha}{2}}}{\sqrt{n}}\right)$$

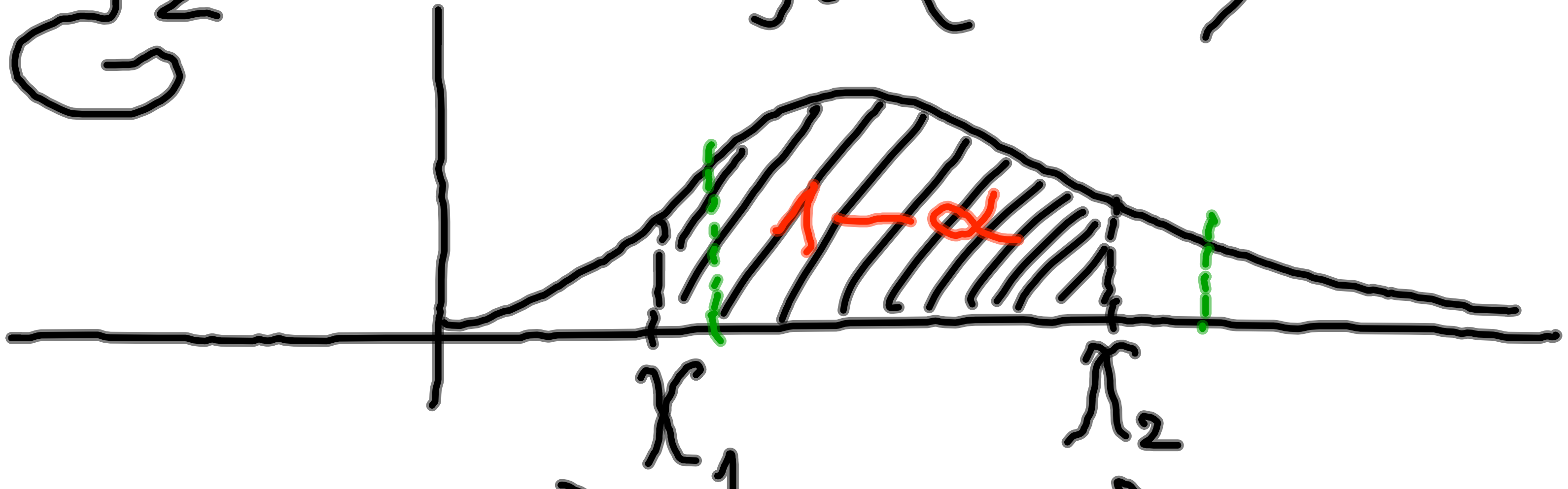
$u_{1-\frac{\alpha}{2}}$ … $(1-\frac{\alpha}{2})$-kvantil $N(0,1)$



Interval spolehlivosti pro rozptyl normálního rozdělení

μ neznáme

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$



$$P\left(\underbrace{\chi_1 < \frac{(n-1)S^2}{\boxed{\sigma^2}} < \chi_2}_{}\right) = 1-\alpha$$

$\chi_1 = \chi^2_{\frac{\alpha}{2}}(n-1)$

$\chi_2 = \chi^2_{1-\frac{\alpha}{2}}(n-1)$

$$P\left(\underbrace{\chi_1 < \frac{(m-1)S^2}{\sigma^2}} < \chi_2\right) = 1-\alpha$$

$$\sigma^2 < \frac{(m-1)S^2}{\chi_1} \qquad \frac{(m-1)S^2}{\chi_2} < \sigma^2$$

$$\frac{(m-1)S^2}{\chi_2} < \sigma^2 < \frac{(m-1)S^2}{\chi_1}$$

$$\boxed{P\left(\frac{(m-1)S^2}{\chi_2} < \sigma^2 < \frac{(m-1)S^2}{\chi_1}\right) = 1-\alpha}$$

Pro rozptyl

$$I_{1-\alpha} = \left(\frac{(m-1)S^2}{\underbrace{\chi_2}_{}} , \frac{(n-1)S^2}{\chi_1} \right)$$

$$\sigma = \sqrt{\sigma^2}$$

$\alpha = 0,05$

$$\chi^2_{0,025}(10) = 3,25$$

α

$$\chi^2_{\frac{\alpha}{2}}(\underline{n-1})$$

$$\chi^2_{1-\frac{\alpha}{2}}(n-1)$$