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Jan Melecký

A dynamics of intrinsic value in simple stock market models

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Mathematical Analysis

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Jan Melecký

DYNAMIKA VNITŘNÍ HODNOTY V JEDNODUCHÝCH MODELECH AKCIOVÉHO TRHU

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Dizertant:	Ing. Jan Melecký Matematický ústav v Opavě Slezská univerzita v Opavě
Školitel:	Doc. RNDr. Kristína Smítalová, CSc. Matematický ústav v Opavě Slezská univerzita v Opavě
Školící pracoviště:	Matematický ústav v Opavě Slezská univerzita v Opavě
Oponenti:	Prof. RNDr. Miroslav Bartušek, DrSc. Přírodovědecká fakulta Masarykova univerzita
	Prof. RNDr. Jaroslav Ramík, CSc. Obchodně podnikatelská fakulta v Karviné Slezská univerzita v Opavě

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Předseda zkušební komise:	Prof. RNDr. Jaroslav Smítal, DrSc. Matematický ústav v Opavě Slezská univerzita v Opavě Na Rybníčku 1 746 01

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Supervisor: Doc. RNDr. Kristína Smítalová, CSc.
Mathematical Institute in Opava
Silesian University in Opava
Na Rybníčku 1
746 01 Opava
Czech Republic

Contents

1 Introduction

The submitted Thesis deals with intrinsic value of stock dynamics. The well known heterogeneous agent models with fundamentalists and chartists by Farmer and Joshi [11] and Chiarella [5] are added with a simple intrinsic value of stock dynamics. This dynamics is described by a linear differential and/or difference equation in the adopt model by Farmer and Joshi and a nonlinear differential equation in the adopt model by Chiarella. The assumptions for these approaches, the description of models and the analysis of the stationary points and their local stability are introduced in three papers that belong to this Thesis. The comparison of the model data with the behavior of the stock price time series for the General Motors and Microsoft stocks at NYSE is included in [18] (the second appendix). The results on the local stability analysis of the stationary point are compared with the ones obtained by Chiarella, see [19] (the third appendix). The Thesis is organized as follows. In section 2 we recall the brief history of economic dynamics. In section 3 we inform on the intrinsic value of stock meaning and its conception in heterogeneous agent models. In section 4 we summarize the main obtained results. Section 5 contains a brief discussion of the obtained results.

2 Main Results

Let us summarize the main results of this Thesis that are in detail described in its appendices. In the paper [17] (*the first appendix*) we consider the following system of differential equations with delay

$$\begin{aligned} \dot{P}(t) &= k_1(F(t) - P(t)) + k_2(P(t) - P(t-h)) \\ \dot{F}(t) &= k_3F(t) + k_4(P(t) - F(t)) \end{aligned} \tag{2.1}$$

where $P(t)$, $F(t)$ denote the market price of stock and the intrinsic value of stock in time t, respectively. The parameter h is the chartists' time delay and k_1, k_2, k_3, k_4 are constants.

The first equation of (2.1) describes the market price of stock dynamics. This equation is similar to the fundamentalist/chartist model by Farmer and Joshi [11]. The only diference is that we consider, contrary to [11], the absolute values of $P(t)$ and $F(t)$ rather then their logarithms. The second equation of (2.1) represents our approach to the intrinsic value of stock time evolution. Our assumptions for this approach are following. We suppose that the change of intrinsic value depends on its current level. That means the company with the higher intrinsic value of stock has better conditions for consequent economic growth and the intrinsic value of its stock increases more quickly. This assumption is described by the term $k_3F(t)$. The second term, $k_4(P(t) - F(t))$, describes the stock *market feedback*. As it is specified in more detail in [19] we consider the following assumptions for this approach. The real stock prices changes are known to affect economy. If the current difference between the stock price and the intrinsic value is positive, then this positive signal of the stock market supports the development of the company in question (for example the position of the company on the primary stock market) and therefore the intrinsic value of stock and vice versa. This assumption is considered more likely for long-time trend. For short-time trend, we suppose that some fundamentalists are influenced by the current price and they revise their assessments of the intrinsic value.

In order to review the stability of the stationary points of (2.1) and the other systems below we use standard theorems, see e.g. [12]. For $k_3 = 0$ system (2.1) has infinitely many stationary points with $\overline{P} = \overline{F}$ where $\overline{F}$ is arbitrary. If $k_3 \neq 0$, then there exists only one stationary point $\overline{P} = \overline{F} = 0$. Consider first $h = 0$.

Proposition 5.1. *If $k_3 = 0$, $h = 0$, then the stationary points $\overline{P} = \overline{F}$ are stable.*

Proposition 5.2. *If $k_3 < 0$, $h = 0$, then the stationary point $\overline{P} = \overline{F} = 0$ is global stable node.*

Proposition 5.3. *If $k_3 > 0$, $h = 0$, then the stationary point $\overline{P} = \overline{F} = 0$ is saddle.*

Now consider the case for infinite h.

Proposition 5.4. *If $k_3 = 0$, $h \to \infty$, then the stationary points $\overline{P} = \overline{F}$ are saddles.*

Proposition 5.5. *If $k_3 \neq 0$, $h \to \infty$, then the stationary point $\overline{P} = \overline{F} = 0$ is globally stable if the conditions*

$$k_1 - k_2 - k_3 + k_4 > 0,$$
$$-k_1k_3 + k_2k_3 - k_2k_4 \geq 0$$

are satisfied.

The assumptions when $h = 0$ or $h \to \infty$ have only theoretical sense. We suppose that the real stock market working takes "adequate" length of time delay h. The result of stability analysis for this case is following.

Proposition 5.6. *For finite $h > 0$ the system (2.1) has no stable stationary points for all choices of parameters.*

The finite-difference version of model (2.1) was examined in conjunction with Artur Sergyeyev and the results are introduced in [18] (*the* *second appendix*). Our model contains the following equations

$$\begin{aligned} P_{n+1} &= P_n + k_1(F_n - P_n) + k_2(P_n - P_{n-m}), \\ F_{n+1} &= F_n + k_3 F_n + k_4(P_n - F_n) \end{aligned} \tag{2.2}$$

where n is the discrete time (number of days), P_n is the stock price at the end of the nth business day (i.e., the closing stock price for the nth day), m is a nonnegative integer describing the delay, F_n is the intrinsic value of stock at the end of the nth business day and k_1, k_2, k_3, k_4 are constant.

The general solution of (2.2) is given by the following formula:

$$P_n = \sum_{i=1}^{m+2} a_i \lambda_i^n, \qquad F_n = \sum_{i=1}^{m+2} b_i \lambda_i^n,$$

where $a_i = b_i(\lambda_i - 1 - k_3 - k_4)/k_4$, and b_i are arbitrary constants. The Cauchy-type problem for (2.2) is set in the following way: having determined the values F_0 and $P_{-m}, P_{-m+1}, \ldots, P_0$ we can calculate P_n and F_n for all natural n. The values of P_n and F_n for $n > 0$ are independent of $F_{-m}, \ldots, F_{-1}$, and therefore the latter need not be included into the Cauchy data.

The stability analysis provides the following results. For $k_1 k_3 \neq 0$ the discrete dynamical system (2.2) has only one stationary point $P_n = F_n = 0$. If $k_1 = 0$ or $k_3 = 0$, then (2.2) has infinitely many stationary points. Only the first situation seems to be realistic and so we dealt with it.

Proposition 5.7. *The stationary point $P_n = F_n = 0$ of (2.2) is not locally stable if $k_1 k_3 \geq 0$ or $|k_2(1 + k_3 - k_4)| \geq 1$.*

The behavior of (2.2) is illustrated with three simulations. The first one shows the evolution of the stock price P and the intrinsic value F for an artificial initial data (Fig.1 in [18]). The other simulations show the comparison of the model data with the real time series for the General Motors and Microsoft stocks at NYSE (Fig.1,2 in [18]). The comparison with the real-world data shows that upon a suitable choice of parameters our model exhibits a behavior reasonably similar to that of the real stock, at least on the qualitative level and for shorter time series.

In the paper [19] (*the third appendix*) we adopt the model with fundamentalists and chartists by Chiarella [5], whose antecedents are Day and Hung [9], Beja and Goldman [1], and Zeeman [22]. We try to extend Chiarella's model with an approach to intrinsic value of stock dynamics. We complete Chiarella's 2-D system of differential equations with the third equation. Chiarella's model [5]

$$\begin{aligned}\dot{P}(t) &= a(W(t)-P(t))+h(\Psi(t)-g(t))\\ \dot{\Psi}(t) &= c(\dot{P}(t)-\Psi(t))\end{aligned}\tag{2.3}$$

is generalized by the equation describing the intrinsic value of stock dynamics. The resulting model is given by the following differential system:

$$\begin{aligned}\dot{P}(t) &= a(W(t)-P(t))+h(\Psi(t)-g(t))\\ \dot{\Psi}(t) &= c(\dot{P}(t)-\Psi(t))\\ \dot{W}(t) &= \alpha k(W(t))+\beta(P(t)-W(t)),\end{aligned}\tag{2.4}$$

where state variables are

$P(t)$ ... logarithm of the market price of stock,
$W(t)$ ... logarithm of the intrinsic value of stock,
$\Psi(t)$ ... the chartists' assessment of the current trend in $P(t)$

and other parameters denote

$g(t)$ ... the return available on investments of alternative securities (e.g. $\Psi(t)$ could refer to stocks and $g(t)$ to public bonds),
h, k ... functions with specific properties,
a, c, α, β ... model parameters.

The assumptions on the intrinsic value dynamics in systems (2.1) and (2.2) there are extended by the function k. This function restricts the influence of the absolute value of W upon its speed of change.

Let us denote $b = h'(-g)$, $d = k'(\overline{W})$, $f = \beta - \alpha d$, and $\tau = 1/c$. Furthermore we denote the bifurcation points of systems (2.3) and (2.4) as $\tau^\star_{CH}$ and $\tau^\star$, respectively. Chiarella [5] has proved (provided that $W(t)$ and $g(t)$ are constants) that the system (2.3) has the stationary point

$$\overline{A}_{CH} = (\overline{P}, \overline{\Psi}) = (W + \frac{h(-g)}{a}, 0)$$

and that $\overline{A}_{CH}$ is locally stable for $\tau > (b-1)/a = \tau^\star_{CH}$. The stability analysis of (2.4) gives the following results. If $g(t)$ is constant, then the system (2.4) has a unique stationary point

$$\overline{A} = \left(\overline{P}, \overline{\Psi}, \overline{W}\right) = \left(k^{-1}\left(\frac{-h(-g)}{a}\frac{\alpha}{\beta}\right) + \frac{h(-g)}{a}, 0, k^{-1}\left(\frac{-h(-g)}{a}\frac{\alpha}{\beta}\right)\right).$$

Proposition 5.8. *For $\alpha > 0$ the stationary point $\overline{A}$ of the system (2.4)*

is unstable.
Proposition 5.9. *Let $b \in (0,1\rangle$ and $\alpha < 0$. Then the stationary point $\overline{A}$ of the system (2.4) is locally stable.*
Proposition 5.10. *Let $b \in (1, b_{kr.})$, $\alpha < 0$ and $\tau > \tau^\star$. Then the stationary point $\overline{A}$ of the system (2.4) is locally stable. The points $b_{kr.}$ and $\tau^\star$ are in form*

$$b_{kr.} = \frac{a+f}{f} \quad \text{and} \quad \tau^\star = \frac{2A}{-B-\sqrt{B^2-4AC}} < \frac{b-1}{a} = \tau^\star_{CH},$$

where

$$\begin{aligned} A &= (b-1)(-a+bf-f) \\ B &= (a+f)^2 + ab\beta - bf^2 - 2abf \\ C &= -a(f+a)(\beta - f). \end{aligned}$$

Proposition 5.11. *Let $b = b_{kr.}$, $\alpha < 0$ and $\tau > \tau^\star = (b-1)/a$. Then the stationary point $\overline{A}$ of the system (2.4) is locally stable.*
Proposition 5.12. *Let $b > b_{kr.}$, $\alpha < 0$ and $\tau > \tau^\star$. Then the stationary point $\overline{A}$ of the system (2.4) is locally stable. The point $\tau^\star$ reads*

$$\tau^\star = \frac{2A}{-B-\sqrt{B^2-4AC}} > \frac{b-1}{a} = \tau^\star_{CH}.$$

The parameter b influences the relation between $\tau^\star$ and $\tau^\star_{CH}$. Provided that $b = b_{kr.} = (a+f)/f$, we have $\tau^\star = \tau^\star_{CH}$. If $b < b_{kr.}$ ($b > b_{kr.}$), then $\tau^\star < \tau^\star_{CH}$ ($\tau^\star > \tau^\star_{CH}$). That means the value interval of the parameter τ making instability of $\overline{A}$ is being smaller (larger) for $b < b_{kr.}$ ($b > b_{kr.}$) comparing that to the stationary point $\overline{A}_{CH}$ of system (2.3).

By using the additive state variables $\xi(t) = P(t) - W(t)$ we reduce the model (2.4) to the following system

$$\begin{aligned} \dot{\xi}(t) &= -(a+\beta)\xi(t) + h(\Psi(t) - g) \\ \dot{\Psi}(t) &= -ac\xi(t) - c\Psi(t) + ch(\Psi(t) - g). \end{aligned} \tag{2.5}$$

For the local stability analysis we have the simplifying assumptions $g(t) \equiv const.$ and $\alpha = 0$. We show that with these assumptions the system (2.5) can have one, two or three stationary points. For the selected model parameter $l = h'(\overline{\Psi} - g)$ we give conditions that determine the quality of stationary points. The stationary point $(\overline{\xi}, \overline{\Psi})$ is *locally stable* if and only

if $trA < 0$ and $detA > 0$ that means $l < (a+\beta)/c+1$ and simultaneously $l < a/\beta + 1$.
The stationary point $(\overline{\xi}, \overline{\Psi})$ is *unstable node* or *unstable focus* if and only if $trA > 0$ and $detA > 0$ that means $l > (a+\beta)/c+1$ and simultaneously $l < a/\beta + 1$.
Now, we can state the following propositions.
Proposition 5.13. *If* $(a+\beta)/c < a/\beta$, *then*
(i) for $l < (a+\beta)/c+1$, *the stationary point* $(\overline{\xi}, \overline{\Psi})$ *is locally asymptotic stable,*
(ii) for $(a+\beta)/c+1 < l < a/\beta+1$, *it is unstable,*
(iii) and for $l > a/\beta+1$, *it is saddle.*
Proposition 5.14. *If* $a/\beta < (a+\beta)/c$, *then*
(i) for $l < a/\beta+1$, *the stationary point* $(\overline{\xi}, \overline{\Psi})$ *is locally asymptotic stable,*
(ii) and for $l > a/\beta+1$, *it is saddle.*
The geometric analysis illustrates the state variables motion at the phase plane. Very interesting situation arises for $l = l_0 = (a+\beta)/c+1$. All trajectories of (2.5) are closed in the neighbourhood of the stationary point $(\overline{\xi}, \overline{\Psi})$ in this case. That means the difference of the price $P(t)$ and the intrinsic value $W(t)$ oscillates. However using the well known the Bendixson theorem it is possible to show that the system (2.5) has no isolated closed trajectory.

3 Conclusions

A simple intrinsic value of stock dynamics is included into three stock market models. These models contain fundamentalist and chartist trading strategies and generalize the well known models by Farmer and Joshi and Chiarella. The approach to the intrinsic value of stock is different from what is typically found in the earlier literature, where the intrinsic value of stock is considered either as constant or a stochastic variable. Our approach is founded on the following two assumptions. Firstly, the change of intrinsic value depends on its current level and this effect is restricted by s-shaped function. Secondly, we suppose the *market feedback* which we perceive as follows. The positive signal of stock market when the stock price is larger than its intrinsic value supports the development of the company in question (at least the position of the company on the primary stock market) and therefore the intrinsic value of its stock and vice versa. This assumption is considered more likely for long-time trend. For short-time trend, we suppose that some fundamentalists are influenced by the current price of stock and they revise their assessments of intrinsic value.

The stability analysis of the stationary points is given for all proposed models. The results show that the application of our approach to intrinsic value dynamics supports the volatile behavior of stock price which is the typical property of the real stock price time series. The comparison of model data with the real stock price time series for the General Motors and Microsoft at NYSE indicate that upon a suitable choice of parameters our model exhibits a behavior reasonably similar to that of the real stock, at least on the qualitative level and for shorter time series.

In future we intend to continue at work on the generalized Chiarella's model. Our aim is to examine the influence of the other model parameters on the model behavior. The subject to discuss can be also the finite-difference version of this model.

4 Publications

[1] Melecký, J., *A model of stock prices behavior*, Preprint Series in Mathematical Analysis, Mathematical Institute in Opava, Silesian University in Opava, MA 44/2004.

[2] Melecký, J., *A model of stock prices behavior*, Proc. 4th International Conference APLIMAT, Slovak University of Technology in Bratislava, Bratislava 2005, 461-466.

[3] Melecký, J., *A Simple Stock Market Model Involving Delay*, Bulletin of the Czech Econometric Society, **23**, 2006, 37-45.

[4] Melecký, J., Sergyeyev, A., *A Simple finite-difference stock market model involving intrinsic value*, Chaos, Solitons and Fractals, (in print).

[5] Melecký, J., *A simple stock market model involving intrinsic value dynamics*, Preprint Series in Mathematical Analysis, Mathematical Institute in Opava, Silesian University in Opava, Preprint MA 60/2007.

5 Presentations

[1] 4th International Conference APLIMAT, Slovak University of Technology in Bratislava, Slovak Republic, February 2005. Talk on [2].

[2] 24th International Conference Mathematical Methods in Economics, Pilsen, Czech Republic, September 2006. Talk on [3].

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